

**Faculty of Science Course Syllabus – updated Sept 2019****Department of Mathematics & Statistics****MATH 4500 01****Introduction to Harmonic Analysis****Fall 2019/2020****Instructor(s):** Keith Taylor    keith.taylor@dal.ca    Chase 123**Lectures:**        MWF 8:35-9:25        Chase 319**Office Hours:**    MWF 9:30-11:30 in Chase 123

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**Course Description**

(Calendar Description) This course covers the basic elements of  $L^p$ -spaces, convolution, interpolation, maximal functions, Fourier analysis of functions, and the theory of generalized functions, or distributions. Further topics may include  $L^2$ -Sobolev spaces, boundary values of harmonic functions, spherical harmonics, singular integral operators, or multipliers. (See the detailed overview for this semester's topics.)

**Course Prerequisites**

MATH 4010.03

**Course Objectives/Learning Outcomes**

Students will develop a basic understanding of various aspects of harmonic analysis with a focus on harmonic analysis on Euclidean space, but with an introduction to the Finite Fourier Transform and the representation theory of some important non-abelian groups.

**Course Assessment**

Assessment is based on a Final Exam assignments.

<b>Component</b>	<b>Weight (% of final grade)</b>	<b>Date</b>
<b>Final exam</b>	<b>50%</b>	<b>To be determined</b>
<b>Assignments</b>	<b>50%</b>	<b>Due approximately every second week</b>

**Textbook**

There is no assigned textbook for this course. We will make use of basic results from Lebesgue integration theory. A very good source for this is *Measure, Integration & Real Analysis* by Sheldon Axler. The author makes an e-version available for free at <http://measure.axler.net/MIRA.pdf>. Chapter 11 contains a brief treatment of Fourier Analysis. Most of the content in this course will be delivered in the lectures.

**Conversion of numerical grades to Final Letter Grades follows the Dalhousie Common Grade Scale**

A+ (90-100)	B+ (77-79)	C+ (65-69)	D (50-54)
A (85-89)	B (73-76)	C (60-64)	F (<50)
A- (80-84)	B- (70-72)	C- (55-59)	

**Course Policies**

Late assignments will not be accepted without a valid reason. A make-up final exam will be available to those who miss for medical reasons.

**Course Content**

Harmonic Analysis refers to representing functions as sums of simpler functions by exploiting any algebraic structure (for example addition and multiplication of real numbers) in the space of the independent variables for the functions. This course will introduce the general approach through the development of several different versions of harmonic analysis that, on the surface, seem to be unrelated. The major emphasis will be on the Fourier transform on  $\mathbf{R}^n$ ,  $n$ -dimensional Euclidean space, and developing it as a tool for applications such as the study of PDEs, signal and image analysis or quantum mechanics.

## Outline:

1. (2 hours) Introduction and calculation of characters of important groups.
  2. (3 hours) Finite Fourier Transform and the Fast Fourier Transform (FFT), arguably the most important algorithm for numerical calculations.
  3. (6 hours) Fourier series: Periodic functions,  $L^2[0; 1]$  and  $L^2(\mathbf{T})$ , where  $\mathbf{T}$  is the circle group. An orthonormal basis of  $L^2[0; 1]$  and Fourier series of a convolution.
  4. (20 hours) Fourier analysis on  $\mathbf{R}^n$ 
    - Review of properties of  $L^p(\mathbf{R}^n)$  spaces and their duality.
    - $L^1(\mathbf{R}^n)$  as a Banach  $*$ -algebra under convolution.
    - $L^2(\mathbf{R}^n)$  as a Hilbert space. When  $n = 1$ ,  $L^2(\mathbf{R})$  models the space of finite energy signals. When  $n = 2$ , it models the spaces of images for image analysis.
    - Basic definition and computational properties of the Fourier transform.
    - The Schwartz space,  $S(\mathbf{R}^n)$ , and the Fourier transform as a bijection on  $S(\mathbf{R}^n)$ .
    - Tempered distributions and the Fourier transform.
    - The Paley - Wiener Theorem.
    - The Fourier transform as a unitary operator on  $L^2(\mathbf{R}^n)$ . (Plancherel's Theorem)
    - The Poisson summation formula.
    - An Interpolation Theorem.
    - Fourier transform on  $L^p(\mathbf{R}^n)$  and the Hausdorff-Young inequality.
  5. (4 hours) A non-commutative group. Harmonic analysis on the group of affine transformations of  $\mathbf{R}$  and the Continuous Wavelet Transform.
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## University Policies and Statements

**This course is governed by the academic rules and regulations set forth in the University Calendar and by Senate**

### Academic Integrity

At Dalhousie University, we are guided in all of our work by the values of academic integrity: honesty, trust, fairness, responsibility and respect (The Center for Academic Integrity, Duke University, 1999). As a student, you are required to demonstrate these values in all of the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity.

**Information:** [https://www.dal.ca/dept/university\\_secretariat/academic-integrity.html](https://www.dal.ca/dept/university_secretariat/academic-integrity.html)

### Accessibility

The Advising and Access Services Centre is Dalhousie's centre of expertise for student accessibility and accommodation. The advising team works with students who request accommodation as a result of a disability, religious obligation, or any barrier related to any other characteristic protected under Human Rights legislation (Canada and Nova Scotia).

**Information:** [https://www.dal.ca/campus\\_life/academic-support/accessibility.html](https://www.dal.ca/campus_life/academic-support/accessibility.html)

### Student Code of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner—perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution.

**Code:** [https://www.dal.ca/dept/university\\_secretariat/policies/student-life/code-of-student-conduct.html](https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html)

### Diversity and Inclusion – Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness

**Statement:** <http://www.dal.ca/cultureofrespect.html>

### Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel and support. Visit or e-mail the Indigenous Student Centre (1321 Edward St) ([elders@dal.ca](mailto:elders@dal.ca)).

**Information:** [https://www.dal.ca/campus\\_life/communities/indigenous.html](https://www.dal.ca/campus_life/communities/indigenous.html)

### Important Dates in the Academic Year (including add/drop dates)

[https://www.dal.ca/academics/important\\_dates.html](https://www.dal.ca/academics/important_dates.html)

### University Grading Practices

[https://www.dal.ca/dept/university\\_secretariat/policies/academic/grading-practices-policy.html](https://www.dal.ca/dept/university_secretariat/policies/academic/grading-practices-policy.html)

### Missed or Late Academic Requirements due to Student Absence (policy)

[https://www.dal.ca/dept/university\\_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html](https://www.dal.ca/dept/university_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html)

## Student Resources and Support

### Advising

**General Advising** [https://www.dal.ca/campus\\_life/academic-support/advising.html](https://www.dal.ca/campus_life/academic-support/advising.html)

**Science Program Advisors:** <https://www.dal.ca/faculty/science/current-students/academic-advising.html>

**Indigenous Student Centre:** [https://www.dal.ca/campus\\_life/communities/indigenous.html](https://www.dal.ca/campus_life/communities/indigenous.html)

**Black Students Advising Centre:** [https://www.dal.ca/campus\\_life/communities/black-student-advising.html](https://www.dal.ca/campus_life/communities/black-student-advising.html)

**International Centre:** [https://www.dal.ca/campus\\_life/international-centre/current-students.html](https://www.dal.ca/campus_life/international-centre/current-students.html)

### **Academic supports**

**Library:** <https://libraries.dal.ca/>

**Writing Centre:** [https://www.dal.ca/campus\\_life/academic-support/writing-and-study-skills.html](https://www.dal.ca/campus_life/academic-support/writing-and-study-skills.html)

**Studying for Success:** [https://www.dal.ca/campus\\_life/academic-support/study-skills-and-tutoring.html](https://www.dal.ca/campus_life/academic-support/study-skills-and-tutoring.html)

**Copyright Office:** <https://libraries.dal.ca/services/copyright-office.html>

**Fair Dealing Guidelines** <https://libraries.dal.ca/services/copyright-office/fair-dealing.html>

### **Other supports and services**

**Student Health & Wellness Centre:** [https://www.dal.ca/campus\\_life/health-and-wellness/services-support/student-health-and-wellness.html](https://www.dal.ca/campus_life/health-and-wellness/services-support/student-health-and-wellness.html)

**Student Advocacy:** <https://dsu.ca/dsas>

**Ombudsperson:** [https://www.dal.ca/campus\\_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html](https://www.dal.ca/campus_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html)

### **Safety**

**Biosafety:** <https://www.dal.ca/dept/safety/programs-services/biosafety.html>

**Chemical Safety:** <https://www.dal.ca/dept/safety/programs-services/chemical-safety.html>

**Radiation Safety:** <https://www.dal.ca/dept/safety/programs-services/radiation-safety.html>

**Scent-Free Program:** <https://www.dal.ca/dept/safety/programs-services/occupational-safety/scent-free.html>