

**Faculty of Science Course Syllabus**  
**Department of Mathematics and Statistics**  
**MATH 3790**  
**Mathematical Problem Solving**  
**Techniques and Methods**  
**Fall 2019**

**Instructor(s):** *Dorette Pronk*      *pronkd@dal.ca*      *Chase 302*

**Lectures:**      *M 1:05 – 2:25*      *LSC – C234*  
                    *W 2:35 – 3:55*      *LSC – P4263*

**Office Hours:** *M 2:30 – 4:00*    *R 12:00 – 1:30* or by appointment

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### **Course Description**

This class will provide an introduction to techniques for solving mathematical problems of the sort encountered in competition (such as the Mathematical Olympiad or the William Putnam competition). There will be self-contained modules developing techniques from several branches of mathematics including number theory, combinatorics, geometry and analysis. The majority of the class time, however, will be devoted to examining examples. Students will be expected to prepare and present in class solutions to assigned problems.

### **Course Prerequisites**

MATH 1000.03, MATH 1010.03 or equivalent, or consent of instructor.

### **Course Objectives/Learning Outcomes**

- Learn techniques for approaching competition problems as well as real life modeling problems and mathematical research problems.
- Learn to work effectively in small groups.
- Coach others to solve problems.
- If there is time and interest: use mathematical models to analyse real world problems and communicate the results (considering several examples).
- Learn to communicate mathematical results with a variety of audiences.
- Analyse solutions by others and learn to spot flaws or gaps in their arguments.
- Learn techniques for tightening mathematical arguments.
- Specific techniques:
  - Proofs: argument by contradiction, mathematical induction, pigeon hole principle, extremal cases, invariants
  - Algebra: clever use of polynomials, results about matrices, famous inequalities and some abstract algebra
  - Calculus: sequences, series, integrals, Stokes' Theorem and functional equations
  - Geometry: clever calculations of areas, useful results about circles, lines and triangles, and some handy three dimensional facts.

- Number Theory: modular arithmetic and Diophantine equations
- Combinatorics: Euler's formula for graphs, combinatorial geometry, counting methods and tricks for calculating probabilities.

### Course Materials

Textbook: R. Gelca, T. Andreescu, *Putnam and Beyond, Second Edition*, Springer Verlag, 2017

I will also make additional notes available through the course website on Brightspace.

A useful book to get a different explanation for some of the techniques is: Paul Zeitz, *The Art and Craft of Problem Solving*, John Wiley and Sons, 1999

### Course Assessment

- Assignments: Weekly assignments will be posted on Brightspace. They will generally be due on Wednesday in class, but can be handed in until 4 PM on Thursday without a penalty. If they are later points will be subtracted for late submission.
- Class participation and preparation: for some classes I will ask you to look at one or two problems and think of one or two ways to approach that problem before the next class (without necessarily solving it). Class attendance is mandatory for this course and class participation will be part of the course grade.
- Leadership and communication: assist and teach in the Dalhousie Math Club and participate in group feedback (or another venue mutually agreed upon by the student and the instructor).
- The midterm will be on October 23, in class.

Component	Weight (% of final grade)	Date
<i>Class Participation</i>	5%	<i>every class</i>
<i>Final exam</i>	40%	<i>scheduled by the registrar</i>
<i>Assignments</i>	15%	<i>weekly on Wednesdays</i>
<i>Math Club Leadership</i>	20%	<i>four nights to be signed up for</i>
<i>Midterm</i>	20%	<i>October 24</i>

### Conversion of numerical grades to Final Letter Grades follows the Dalhousie Common Grade Scale

A+ (90-100)	B+ (77-79)	C+ (65-69)	D	(50-54)
A (85-89)	B (73-76)	C (60-64)	F	(<50)
A- (80-84)	B- (70-72)	C- (55-59)		

### Course Policies

- My office hours are Monday 2:30-4:00 PM and Thursday 12-1:30 PM. You may use office hours for any type of questions you have in regard to the course, including requests for hints or help on assignment questions.

- You are allowed to collaborate on solving the assignment questions, but not on the write-up. So I expect your submitted work to look distinct.
- If you are not able to come to class or submit homework on time, please email the instructor.
- **Leadership Component:** The math department offers currently two math clubs, on Monday from 5:30 till 7:30 PM for junior high and high school students and on Wednesday from 4:30 till 5:30 PM for elementary school students. Students enrolled in this course are asked to attend one of these clubs at least four times: at least twice to observe and participate as classroom aide and at least twice to present a problem solving technique and help participants become familiar with it. After each visit students are required to prepare a reflective report on the club, their own participation and the participation of their class mates, using the questions provided in the forms found on Brightspace. The material covered in the math clubs is taken from old math contests, the Julia Robinson Festival, and training materials provided by the instructor. However, any topic chosen by the presenter and presented at the right level is welcome! A week before attending math club as a presenter, the student needs to meet with Dr. Pronk to choose a topic to present. When the student is presenting material they also need to prepare a hand-out for the students in the club. The first draft of the hand-out needs to be discussed with Dr. Pronk by Thursday for the Monday Club and by Monday for the Wednesday Club. The final version needs to be submitted to Dr. Pronk for photocopying by the morning of the day of the club.

### Course Content (Dates are Approximate)

- September 4: Proof Writing and Problem Solving Strategies: Induction and Pigeon Hole
- September 9&11: Proof Writing and Problem Solving Strategies: Extremal Elements and (Semi-)Invariants
- September 16&18: Algebra (problems that can be solved using polynomials and matrix manipulation)
- September 23&25: Geometry – clever coordinates and complex numbers to the rescue
- September 30 & October 2: Number Theory: Fermat’s Little Theorem, Wilson’s Theorem and related material
- October 7&9: Calculus: sequences, series and recursion
- October 16 & 21: Combinatorics (graph theory, methods for counting and a connection with geometry)
- October 23: **Midterm**
- October 28 & 30: Algebra (further linear algebra results, and inequalities)
- October 31 and November 2: Geometry - further tricks and standard results in 2- and 3-dimensional geometry
- November 4&6: Combinatorics: More Counting Strategies and Probability
- November 11&13: Reading week (no classes)
- November 18&20: Cross-over Tactics: using results from one area in another area
- November 25&27: Number Theory (Diophantine equations) and Calculus (clever tricks with integrals and symmetry of functions)
- December 2&3: A Mixture of Problems – Exam Review

## University Policies and Statements

**This course is governed by the academic rules and regulations set forth in the University Calendar and by Senate**

### Academic Integrity

At Dalhousie University, we are guided in all of our work by the values of academic integrity: honesty, trust, fairness, responsibility and respect (The Center for Academic Integrity, Duke University, 1999). As a student, you are required to demonstrate these values in all of the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity.

**Information:** [https://www.dal.ca/dept/university\\_secretariat/academic-integrity.html](https://www.dal.ca/dept/university_secretariat/academic-integrity.html)

### Accessibility

The Advising and Access Services Centre is Dalhousie's centre of expertise for student accessibility and accommodation. The advising team works with students who request accommodation as a result of a disability, religious obligation, or any barrier related to any other characteristic protected under Human Rights legislation (Canada and Nova Scotia).

**Information:** [https://www.dal.ca/campus\\_life/academic-support/accessibility.html](https://www.dal.ca/campus_life/academic-support/accessibility.html)

### Student Code of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner—perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution.

**Code:** [https://www.dal.ca/dept/university\\_secretariat/policies/student-life/code-of-student-conduct.html](https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html)

### Diversity and Inclusion – Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness

**Statement:** <http://www.dal.ca/cultureofrespect.html>

### Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel and support. Visit or e-mail the Indigenous Student Centre (1321 Edward St) ([elders@dal.ca](mailto:elders@dal.ca)).

**Information:** [https://www.dal.ca/campus\\_life/communities/indigenous.html](https://www.dal.ca/campus_life/communities/indigenous.html)

### Important Dates in the Academic Year (including add/drop dates)

[https://www.dal.ca/academics/important\\_dates.html](https://www.dal.ca/academics/important_dates.html)

### University Grading Practices

[https://www.dal.ca/dept/university\\_secretariat/policies/academic/grading-practices-policy.html](https://www.dal.ca/dept/university_secretariat/policies/academic/grading-practices-policy.html)

### Missed or Late Academic Requirements due to Student Absence (policy)

[https://www.dal.ca/dept/university\\_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html](https://www.dal.ca/dept/university_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html)

## Student Resources and Support

### Advising

**General Advising** [https://www.dal.ca/campus\\_life/academic-support/advising.html](https://www.dal.ca/campus_life/academic-support/advising.html)

**Science Program Advisors:** <https://www.dal.ca/faculty/science/current-students/academic-advising.html>

**Indigenous Student Centre:** [https://www.dal.ca/campus\\_life/communities/indigenous.html](https://www.dal.ca/campus_life/communities/indigenous.html)

**Black Students Advising Centre:** [https://www.dal.ca/campus\\_life/communities/black-student-advising.html](https://www.dal.ca/campus_life/communities/black-student-advising.html)

**International Centre:** [https://www.dal.ca/campus\\_life/international-centre/current-students.html](https://www.dal.ca/campus_life/international-centre/current-students.html)

### Academic supports

**Library:** <https://libraries.dal.ca/>

**Writing Centre:** [https://www.dal.ca/campus\\_life/academic-support/writing-and-study-skills.html](https://www.dal.ca/campus_life/academic-support/writing-and-study-skills.html)

**Studying for Success:** [https://www.dal.ca/campus\\_life/academic-support/study-skills-and-tutoring.html](https://www.dal.ca/campus_life/academic-support/study-skills-and-tutoring.html)

**Copyright Office:** <https://libraries.dal.ca/services/copyright-office.html>

**Fair Dealing Guidelines** <https://libraries.dal.ca/services/copyright-office/fair-dealing.html>

### Other supports and services

**Student Health & Wellness Centre:** [https://www.dal.ca/campus\\_life/health-and-wellness/services-support/student-health-and-wellness.html](https://www.dal.ca/campus_life/health-and-wellness/services-support/student-health-and-wellness.html)

**Student Advocacy:** <https://dsu.ca/dsas>

**Ombudsperson:** [https://www.dal.ca/campus\\_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html](https://www.dal.ca/campus_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html)

### Safety

**Biosafety:** <https://www.dal.ca/dept/safety/programs-services/biosafety.html>

**Chemical Safety:** <https://www.dal.ca/dept/safety/programs-services/chemical-safety.html>

**Radiation Safety:** <https://www.dal.ca/dept/safety/programs-services/radiation-safety.html>

**Scent-Free Program:** <https://www.dal.ca/dept/safety/programs-services/occupational-safety/scent-free.html>