MATH/CSCI 2113: Discrete Structures II Winter 2019

Neil J. Ross
Email: neil.jr.ross@dal.ca
Office: Chase Building room 213
Office hours: Tues. 3:30–5:00 & Wed. 2:30–4:00, or by appointment

Lectures: Time: Mon., Wed. & Fri., 11:35–12:25 Place: Kenneth C. Rowe Management Building 1011 Web presence: BrightSpace

Course Description: This class continues the exploration of discrete mathematics. It aims to develop logical reasoning skills relevant to computer science and mathematics. Elementary counting techniques lead to sophisticated methods to approach combinatorial problems. Methods include bijective counting and discrete probability. Structures such as partitions, permutations, symmetry groups, graphs and trees are introduced. Discrete principles are applied to compression and error-correction of digital information.

Prerequisites: MATH/CSCI 2112 (Discrete Structures I).

Course Objectives/Learning Outcomes:

- Be able to count combinatorial objects using elementary techniques.
- Be able to read a description of a counting problem and derive a recurrence relation to solve the problem.
- Recognize common counting-related sequences such as Fibonacci numbers, Catalan numbers and Stirling numbers, and know the definition of these sequences.
- Be able to compose and manipulate generating functions, and use generating functions to obtain a direct formula for a recursively derived sequence.
- Understand discrete probability spaces and random variables. Be able to compute the expected value of a discrete random variable by expressing it as a sum of relevant indicator variables and using linearity of expectation.
- Be able to define and recognize partitions, permutations, groups, graphs and trees. Know the difference between labelled and unlabelled graphs and trees, and ordered and unordered trees.
- Be able to find the multiplicative subgroups of the integers modulo a given integer.
- Be able to define Shannon entropy and describe what it represents.
- Know the relationship between entropy and compressibility. Be able to find the optimally compressed representation of a random binary channel using Huffman codes.
- Know the basic concepts of error-correction in binary channels. Be able to encode and decode messages with a linear code. Be able to construct Hamming codes.
- Be able to distinguish between countable and uncountable sets. Know Cantor's diagonalization argument.
- Know how to read and understand a short combinatorial argument and reproduce it in their own words.

Course Materials: There is no required text for this course. Notes and pointers to various resources will be provided on Brightspace.

Recommended texts (links are on Brightspace):

- *Discrete Mathematics, and Open Introduction,* by Oscar Levin.
- The Book of Proof, by Richard Hammack. Especially Chapter 3, "Counting".
- *Principles and Techniques of Combinatorics,* by Chen Chuang-Chong and Koh Khee-Meng.
- *Abstract Algebra: Theory and Applications,* by Thomas W. Judson.
- Elementary Probability, by David A. Santos.
- *A student's guide to Coding and Information Theory,* by Stephan M. Moser and Po-Ning Chen.

For additional reading (available in Killam library):

- *Pearls of Discrete Mathematics*, by Martin Erickson.
- Discrete and Combinatorial Mathematics, by Ralph P. Grimaldi.

Assessment:

Assignments. Weekly assignments will be posted on BrightSpace and must be handed in at the *beginning* of class on the due date.

Exams. Two term tests will be held in class, the first on Fri., **Feb. 8**, and the second on Fri., **Mar. 15**. The final examination will be held during the exam period and will be scheduled by the registrar.

Project. Students will be asked to prepare a short report (1–2 pages) on a book/class problem.

Mark Breakdown.	Assignments	20%	Class project	10%
	Term tests	$40\% (= 2 \times 20\%)$	Final exam	30%

Conversion of numerical grades to final letter grades follows the Dalhousie Common Grade Scale:

A + (90–100)	B + (77–79)	C + (65–69)	D (50–54)
A (85–89)	B (73–76)	C (60–64)	F (< 50)
A - (80–84)	B - (70–72)	C - (55-59)	

Policies: (i) Points will be taken off for handing in assignments late; no assignments will be accepted more than 48 hours late. (ii) All tests and exams are closed book: no calculators; no listening devices; switch off cellular phones; bring a pen to print your name on your paper. (iii) A missed exam counts as zero unless under legitimate and reasonably documented circumstances such as illness with a doctor's note; for accommodation, the student must contact me prior to the exam and make arrangements as soon as possible.

Topics:

- (1) Advanced counting. Dealing with repetitions. Inclusion/exclusion revisited.
- (2) Symmetry. Introduction to groups.
- (3) Relations, functions. One-to-one and onto. Using Bijections to count.
- (4) Bijections between infinite sets. Countable and uncountable sets.
- (5) Modelling combinatorial problems.
- (6) Review of recurrence relations. Linear recurrences. Generating functions. Asymptotics.
- (7) Discrete probability. Random variables. Expectation. Indicator variables and the linearity of expectation.
- (8) Graphs and trees—various ways of counting trees.
- (9) Counting and compression. Entropy. Huffman codes. Information theory.
- (10) Binary channels and channel capacity. Error-correcting codes. Hamming codes. Cyclic codes.
- (11) Turing machines. Computability.