

Faculty of Science Course Syllabus Department of Mathematics & Statistics MATH 4055/5055: Advanced Algebra II (Winter 2025)

Instructor: Theo Johnson-Freyd, theo.jf@dal.ca.

Lectures: Tuesdays and Thursdays, 10:05-11:25 Atlantic Time, Chase 227.

Office Hours: Wednesdays, 14:35-16:55 Atlantic Time, Chase 214.

Course website: <http://categorified.net/25Winter5055/>.

This course is governed by the academic [rules and regulations](#) set forth in the University Calendar and by the Senate.

Course description

A *field* is a commutative ring with no proper ideals. It follows from the definition that any homomorphism $F \rightarrow E$ of fields is an injection aka an *extension*. This course will study the theory of field extensions, equivalently the category of fields.

Quite remarkably, we will associate to each field F a group $\text{Gal}(F)$ in such a way that the category of field extensions of F is very well approximated by the category of sets with a transitive $\text{Gal}(F)$ -action. (Here "very well approximated" means "up to adding appropriate finiteness conditions".) This allows questions of commutative algebra and module theory to be translated into questions of group theory.

There are various qualitative "sizes" of extensions $F \rightarrow E$. The ones controlled by $\text{Gal}(F)$ are the "smallest", called the *separable* extensions. There are also finite but *purely inseparable* extensions, and we will analyze these — they are an interesting positive-characteristic phenomenon. Finally, there are *purely transcendental* extensions, which have the flavour of modules or vector spaces, including a theory of bases.

Course materials and delivery

The main textbook for the class is:

- **Abstract Algebra** by D. Dummit and R. Foote, chapters 13 and 14.

It is recommended but not required that students own a copy of the textbook. Supplemental material will be drawn from:

- **Algebra** by T.W. Hungerford, chapters V and VI.
- **Algebra** by S. Lang, chapters V, VI, and VII.
- **Galois Theory** by E. Artin, chapters 2 and 3.

Homework, notes, and so on will be available on the [Course Content](#) page.

Lectures will be given in person. (The occasional lecture may be delivered over Zoom.) This is a small class. Although attendance is not mandatory, your presence or absence will certainly be noticed by all participants.

Course Assessment

Homework assignments will be distributed roughly every two weeks. I encourage you to work together to complete your homework assignments. [Studies have shown that social ties are a main predictor of success in STEM classes.](#) Although you are encouraged to work together, the homework you submit must be written by you individually. There are many online resources offering solutions to homework at all levels. If you choose to use such resources, please be cautious: they often provide too detailed an answer, and students can trick themselves into thinking that they understand more than they do by copying those answers.

There will be a two-hour final exam at the end of the semester. Graduate students have the option to instead take the three-hour Algebra Comprehensive Exam Part II, which will consist of the final exam together with one or two additional questions on material from Math 3031. There will not be a midterm.

Each graduate student will also give a presentation (roughly 15 minutes) on a topic related to the course, to be selected in consultation with the instructor. Undergraduate students are invited to give a presentation for extra credit. Some topics people have spoken on in the past: The inverse Galois problem (engineering specific Galois groups); Galois connections in orders logic (on the abstract theory of the Galois correspondence); and Galois theory for graphs (combining Galois theory, combinatorics, and algebraic topology). Presentations will be given throughout the course.

Component weighting

Math 4055:

Homework: 60%.

Final exam: 40%.

Math 5055:

Homework: 50%.

Presentation: 20%.

Final exam: 30%.

Conversion of numerical grades to Final Letter Grades follows the [Dalhousie Common Grade Scale](#). Note that for graduate students, any grade below B- is recorded as an F.

University Policies and Statements

This course is governed by the academic rules and regulations set forth in the University Calendar and by Senate.

Digital communication

As per the University code, email is an authorized means of communication for academic and administrative purposes within Dalhousie. The University will assign all students an official email address. This address will remain in effect while the student remains a student and for one academic term following a student's last registration. This is the only email address that will be used for communication with students regarding all academic and administrative matters. Any redirection of email will be at the student's own risk. Each student is expected to check her or his official email address frequently in order to stay current with Dalhousie communications.

Missed or Late Academic Requirements due to Student Absence

As per Senate decision instructors may not require medical notes of students who must miss an academic requirement, including the final exam, for courses offered during fall or winter 2020-21 (until April 30, 2021). Information on regular policy, including the use of the Student Declaration of Absence can be found here: https://www.dal.ca/dept/university_secretariat/policies/academic/missed-or-late-academic-requirements-due-to-student-absence.html.

Academic Integrity

At Dalhousie University, we are guided in all of our work by the values of academic integrity: honesty, trust, fairness, responsibility and respect (The Center for Academic Integrity, Duke University, 1999). As a student, you are required to demonstrate these values in all of the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity. Information: https://www.dal.ca/dept/university_secretariat/academic-integrity.html

Accessibility

The Advising and Access Services Centre is Dalhousie's centre of expertise for student accessibility and accommodation. The advising team works with students who request accommodation as a result of a disability, religious obligation, or any barrier related to any other characteristic protected under Human Rights legislation (Canada and Nova Scotia). Information: https://www.dal.ca/campus_life/academic-support/accessibility.html

Student Code of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner—perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution. Code: https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html

Diversity and Inclusion – Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness. Statement: <http://www.dal.ca/cultureofrespect.html>

Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel and support. Visit or e-mail the Indigenous Student Centre (1321 Edward St) (elders@dal.ca). Information: https://www.dal.ca/campus_life/communities/indigenous.html

Important Dates in the Academic Year (including add/drop dates)

https://www.dal.ca/academics/important_dates.html

University Grading Practices

https://www.dal.ca/dept/university_secretariat/policies/academic/grading-practices-policy.html

Student Resources and Support

Advising

- General Advising: https://www.dal.ca/campus_life/academic-support/advising.html
- Science Program Advisors: <https://www.dal.ca/faculty/science/current-students/academic-advising.html>
- Indigenous Student Centre: https://www.dal.ca/campus_life/communities/indigenous.html
- Black Students Advising Centre: https://www.dal.ca/campus_life/communities/black-student-advising.html
- International Centre: https://www.dal.ca/campus_life/international-centre/current-students.html

Academic supports

- Library: <https://libraries.dal.ca/>
- Writing Centre: https://www.dal.ca/campus_life/academic-support/writing-and-study-skills.html
- Studying for Success: https://www.dal.ca/campus_life/academic-support/study-skills-and-tutoring.html
- Copyright Office: <https://libraries.dal.ca/services/copyright-office.html>
- Fair Dealing Guidelines: <https://libraries.dal.ca/services/copyright-office/fair-dealing.html>

Other supports and services

- Student Health & Wellness Centre: https://www.dal.ca/campus_life/health-and-wellness/services-support/student-health-and-wellness.html
- Student Advocacy: <https://www.dsu.ca/student-advocacy-service>
- Ombudsperson: https://www.dal.ca/campus_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html

Safety

- Biosafety: <https://www.dal.ca/dept/safety/programs-services/biosafety.html>
- Chemical Safety: <https://www.dal.ca/dept/safety/programs-services/chemical-safety.html>
- Radiation Safety: <https://www.dal.ca/dept/safety/programs-services/radiation-safety.html>
- Scent-Free Program: <https://www.dal.ca/dept/safety/programs-services/occupational-safety/scent-free.html>