# Faculty of Science Course Syllabus
# Department of Mathematics and Statistics
# Math 3070
# Theory of Numbers
# Fall 2024

**Dalhousie University acknowledges that we are in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People and pays respect to the Indigenous knowledges held by the Mi'kmaq People, and to the wisdom of their Elders past and present. The Mi'kmaq People signed Peace and Friendship Treaties with the Crown, and section 35 of the Constitution Act, 1982 recognizes and affirms Aboriginal and Treaty rights. We are all Treaty people.**

**Dalhousie University also acknowledges the histories, contributions, and legacies of African Nova Scotians, who have been here for over 400 years.**

## Course Instructor

| Section | Instructor | E-mail Address | Office Hours |
|---------|-----------|----------------|--------------|
| 01 | Rob Noble | rnoble@dal.ca | MWF 12:30 PM – 1:30 PM in Chase 205 |

## Course Description

Topics include: congruences and residues; elementary properties of congruences, linear congruences, theorems of Fermat, Euler and Wilson, Chinese remainder theorem, quadratic residues, law of quadratic reciprocity, Legendre, Jacobi and Kronecker symbols, arithmetic functions, algebraic fields, algebraic numbers and integers, uniqueness of factorization, elementary properties of ideals, and class number.

### Course Prerequisites

MATH 2040.03 (or MATH 2135.03)

## Course Structure

### Course Delivery

In person (not recorded).

### Lectures

| Section | Days | Time | Location |
|---------|------|------|----------|
| 01 | MWF | 11:35 AM – 12:25 PM | HENRY HICKS ACADEMIC ADMIN 217 |

## Course Materials

- Textbook: Elementary Number Theory, 2nd Edition, by Underwood Dudley ([Dud08]).

- Course notes (as well as all other important course materials) will be made available on Brightspace.

# Assessment

The Final Grade will be computed as the maximum of the grades obtained from the following two schemes:

**Scheme I:**

| Component | Weight (% of final grade) | Date |
|---|---|---|
| Midterm Exam | 30% | In class on October 30, 2024 |
| Final Exam | 45% | (Scheduled by Registrar) |
| Assignments | 25% | Weekly |

**Scheme II:**

| Component | Weight (% of final grade) | Date |
|---|---|---|
| Final Exam | 75% | (Scheduled by Registrar) |
| Assignments | 25% | Weekly |

**Note: It is strongly recommended that you prepare yourself to be graded on the first scheme; the second scheme is included in order to accommodate students who fail to perform up to their ability on the midterm due to circumstances beyond their control.**

Conversion of numerical grades to final letter grades follows the Dalhousie Grade Scale.

| A+: | (90–100) | A: | (85–89) | A-: | (80–84) |
|---|---|---|---|---|---|
| B+: | (77–79) | B: | (73–76) | B-: | (70–72) |
| C+: | (65–69) | C: | (60–64) | C-: | (55–59) |
| D: | (50–54) | F: | (0–49) | | |

# General Course Policies

- Calculators will NOT be allowed during the midterm or the final examination. In fact, only writing utensils (pencils, lead, erasers, pens, white-out) will be allowed.

- Information about the course may be given during class. It is the responsibility of the students to ensure that they are made aware of what occurs during classes.

# Course Policies on Missed or Late Academic Requirements

- Missed midterms or final exams can be made up, at the discretion of the course instructor for illness or equivalent official inability to write at the scheduled time and place. Please contact the instructor for more information or to request permission to write a make-up midterm or final exam.

- Extensions may be provided for assignments for a good reason (as determined by the professor) as long as the solutions have not yet been posted.

# Course Policies related to Academic Integrity

- Students may collaborate on assignments, but must write up their own solutions, in their own words.

- No collaboration is permitted on the midterm nor the final exam.

# Learning Objectives

The main goal of this course is to understand how to solve polynomial congruences modulo arbitrary moduli. This will require the use of most of the material presented in the course, detailed below in the "Course Content" section.

# Course Content

The contents of the provided course notes will be covered. They consist of the majority of the sections of the textbook as well as some additional material:

1. Mathematical Induction and the Least Integer Principle

2. Integers ([Dud08, §1])

3. Unique Factorization ([Dud08, §2])

4. Linear Diophantine Equations ([Dud08, §3])

5. Congruences ([Dud08, §4])

6. Linear Congruences ([Dud08, §5])

7. Fermat's and Wilson's Theorems ([Dud08, §6])

8. The Divisors of an Integer ([Dud08, §7])

9. Perfect Numbers ([Dud08, §8])

10. Euler's Theorem and Function ([Dud08, §9])

11. Primitive Roots ([Dud08, §10])

12. Quadratic Congruences ([Dud08, §11])

13. Quadratic Reciprocity ([Dud08, §12])

14. Pythagorean Triangles ([Dud08, §16])

15. Infinite Descent and Fermat's Conjecture* ([Dud08, §17])

16. Sums of Squares ([Dud08, §18, 19] and [Ser73, Appendix to Ch. 4])

17. $x^2 - Ny^2 = 1$** ([Dud08, §20])

*As of 1994, and due to Andrew Wiles, "Fermat's Conjecture" is now "Fermat's Last Theorem."
**This diophantine equation is known as "Pell's equation."

# University Policies and Statements

### Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel, and support. Visit or email the Indigenous Student Centre at 1321 Edward St or elders@dal.ca. Additional information regarding the Indigenous Student Centre can be found at: https://www.dal.ca/campus_life/communities/indigenous.html

### Internationalization

At Dalhousie, 'thinking and acting globally' enhances the quality and impact of education, supporting learning that is "interdisciplinary, cross-cultural, global in reach, and orientated toward solving problems that extend across national borders." Additional internationalization information can be found at: https://www.dal.ca/about-dal/internationalization.html

# Academic Integrity

At Dalhousie University, we are guided in all our work by the values of academic integrity: honesty, trust, fairness, responsibility, and respect. As a student, you are required to demonstrate these values in all the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity. Additional academic integrity information can be found at: https://www.dal.ca/dept/university_secretariat/academic-integrity.html

# Accessibility

The Student Accessibility Centre is Dalhousie's centre of expertise for matters related to student accessibility and accommodation. If there are aspects of the design, instruction, and/or experiences within this course (online or in-person) that result in barriers to your inclusion, please contact the Student Accessibility Centre (https://www.dal.ca/campus_life/academic-support/accessibility.html) for all courses offered by Dalhousie with the exception of Truro. For courses offered by the Faculty of Agriculture, please contact the Student Success Centre in Truro (https://www.dal.ca/about-dal/agricultural-campus/student-success-centre.html)

# Conduct in the Classroom - Culture of Respect

Substantial and constructive dialogue on challenging issues is an important part of academic inquiry and exchange. It requires willingness to listen and tolerance of opposing points of view. Consideration of individual differences and alternative viewpoints is required of all class members, towards each other, towards instructors, and towards guest speakers. While expressions of differing perspectives are welcome and encouraged, the words and language used should remain within acceptable bounds of civility and respect.

# Diversity and Inclusion - Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness (Strategic Priority 5.2). Additional diversity and inclusion information can be found at: http://www.dal.ca/cultureofrespect.html

# Student Code of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner - perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution. The full Code of Student Conduct can be found at: https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html

# Fair Dealing Policy

The Dalhousie University Fair Dealing Policy provides guidance for the limited use of copyright protected material without the risk of infringement and without having to seek the permission of copyright owners. It is intended to provide a balance between the rights of creators and the rights of users at Dalhousie. Additional information regarding the Fair Dealing Policy can be found at: https://www.dal.ca/dept/university_secretariat/policies/academic/fair-dealing-policy-.html

# Originality Checking Software

The course instructor may use Dalhousie's approved originality checking software and Google to check the originality of any work submitted for credit, in accordance with the Student Submission of Assignments and Use of Originality Checking Software Policy. Students are free, without penalty of grade, to choose an alternative method of attesting to the authenticity of their work and must inform the instructor no later than the last day to add/drop classes of their intent to choose

an alternate method. Additional information regarding Originality Checking Software can be found at: https://www.dal.ca/dept/university_secretariat/policies/academic/student-submission-of-assignments-and-use-of-originality-checking-software-policy-.html

## Student Use of Course Materials

Course materials are designed for use as part of this course at Dalhousie University and are the property of the instructor unless otherwise stated. Third party copyrighted materials (such as books, journal articles, music, videos, etc.) have either been licensed for use in this course or fall under an exception or limitation in Canadian Copyright law. Copying this course material for distribution (e.g. uploading to a commercial third-party website) may lead to a violation of Copyright law.

# Student Resources and Support

## University Policies and Programs

Important Dates in the Academic Year (including add/drop dates): http://www.dal.ca/academics/important_dates.html

Classroom Recording Protocol: https://www.dal.ca/dept/university_secretariat/policies/academic/classroom-recording-protocol.html

Dalhousie Grading Practices Policies: https://www.dal.ca/dept/university_secretariat/policies/academic/grading-practices-policy.html

Grade Appeal Process: https://www.dal.ca/campus_life/academic-support/grades-and-student-records/appealing-a-grade.html

Sexualized Violence Policy: https://www.dal.ca/dept/university_secretariat/policies/health-and-safety/sexualized-violence-policy.html

Scent-Free Program: https://www.dal.ca/dept/safety/programs-services/occupational-safety/scent-free.html

## Learning and Support Resources

General Academic Support - Advising (Halifax): https://www.dal.ca/campus_life/academic-support/advising.html

General Academic Support - Advising (Truro): https://www.dal.ca/about-dal/agricultural-campus/ssc/academic-support/advising.html

Student Health & Wellness Centre: https://www.dal.ca/campus_life/health-and-wellness.html

On Track (helps you transition into university, and supports you through your first year at Dalhousie and beyond): https://www.dal.ca/campus_life/academic-support/On-track.html

Indigenous Student Centre: https://www.dal.ca/campus_life/communities/indigenous.html

Indigenous Connection: https://www.dal.ca/about-dal/indigenous-connection.html

Elders-in-Residence (The Elders in Residence program provides students with access to First Nations elders for guidance, counsel, and support. Visit the office in the Indigenous Student Centre or contact the program at elders@dal.ca or 902-494-6803: https://cdn.dal.ca/content/dam/dalhousie/pdf/academics/UG/indigenous-studies/Elder-Protocol-July2018.

pdf

Black Student Advising Centre: https://www.dal.ca/campus_life/communities/black-student-advising.html

International Centre: https://www.dal.ca/campus_life/international-centre.html

South House Sexual and Gender Resource Centre: https://southhousehalifax.ca/about/

LGBTQ2SIA+ Collaborative: https://www.dal.ca/dept/vpei/edia/education/community-specific-spaces/LGBTQ2SIA-collaborative.html

Dalhousie Libraries: http://libraries.dal.ca/

Copyright Office: https://libraries.dal.ca/services/copyright-office.html

Dalhousie Student Advocacy Services: https://www.dsu.ca/dsas?rq=student%20advocacy

Dalhousie Ombudsperson: https://www.dal.ca/campus_life/safety-respect/student-rights-and-responsibilities/where-to-get-help/ombudsperson.html

Human Rights and Equity Services: https://www.dal.ca/dept/hres.html

Writing Centre: https://www.dal.ca/campus_life/academic-support/writing-and-study-skills.html

Study Skills/Tutoring: http://www.dal.ca/campus_life/academic-support/study-skills-and-tutoring.html

Faculty of Science Advising Support: https://www.dal.ca/faculty/science/current-students/undergrad-students/degree-planning.html

## Safety

Biosafety: http://www.dal.ca/dept/safety/programs-services/biosafety.html

Chemical Safety: https://www.dal.ca/dept/safety/programs-services/chemical-safety.html

Radiation Safety: http://www.dal.ca/dept/safety/programs-services/radiation-safety.html

Laser Safety: https://www.dal.ca/dept/safety/programs-services/radiation-safety/laser-safety.html

# References

[Dud08]  Underwood Dudley, *Elementary number theory*, 2nd ed., Dover Publications, 2008.

[Ser73]   Jean-Pierre Serre, *A course in arithmetic*, Springer, 1973.