

Measure Theory and Integration Syllabus

Department of Mathematics and Statistics MATH 4010/5010 Fall 2023

Dalhousie University acknowledges that we are in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People and pays respect to the Indigenous knowledges held by the Mi'kmaq People, and to the wisdom of their Elders past and present. The Mi'kmaq People signed Peace and Friendship Treaties with the Crown, and section 35 of the Constitution Act, 1982 recognizes and affirms Aboriginal and Treaty rights. We are all Treaty people.

Dalhousie University also acknowledges the histories, contributions, and legacies of African Nova Scotians, who have been here for over 400 years.

Name	Email	Office Hours
Suresh Eswarathasan	sr766936@dal.ca	TBD Chase 316

Course Instructor:

(Associate) Professor Suresh Eswarathasan

Course Description

Lebesgue's theory of measure and integration. The topics include: sigma-algebras, measures, construction of measures, Lebesgue measure on the real line, Borel measures, measurable functions, the Lebesgue integral and convergence theorems, L^p spaces, product measures and the Fubini-Tonelli theorem.

Course Prerequisites: Math 3501



Student Resources

The MATH/STAT Learning Centre is located in Chase 119 and will be operating in-person and remotely. It opens on Sept. 5 and support is available Monday through Friday from 11:30am - 4:30pm and Monday through Friday evenings from 6:30-7:30pm, until Dec. 19. Register for the Brightspace "course" at https://www.dal.ca/faculty/science/math-stats/about/learning-centre.html

Course Structure

Course Delivery: Tentatively, the lectures will be held in-person and will not be recorded. However, all my lecture notes, from which the in-person lectures will be based, will be uploaded at the end of the week.

Lectures: MWF, 11:35pm-12:25pm AST in Killam Library 4106

Tutorials: No tutorials

Course Materials

- In-class notes. Uploaded lecture notes.
- Strongly suggested (but not required) textbook: E. Stein & R. Shakarchi, "Real Analysis: Measure Theory, Integration, and Hilbert Spaces", Princeton University Press (2005).
- Course website on Brightspace is accessed through dal.brightspace.com



Assessment

For 4500 students:

40 % - biweekly homework assignments

10% - biweekly journals - briefly describe the material covered the previous two weeks, how much time was spent on it, the difficulties experienced, and how the issues were ultimately resolved.

25 % - In-class midterm; tentative date: October 27, 2023

25% - Final exam; date TBD in Dalhousie's predetermined exam period.

For 5500 students:

40 % - biweekly homework assignments

10% - biweekly journals - The students are expected to keep a journal (worth %10 of the grade in the class) where they briefly describe the material they looked at on that day, how much time they spent on class material, the difficulties they experienced, and how they ultimately resolved them.

25 % - In-class midterm; tentative date: October 27, 2023

25% - Final exam; date TBD in Dalhousie's predetermined exam period.

Conversion of numerical grades to final letter grades follows the Dalhousie Grade Scale

	<u> </u>	<u> </u>	
A+ (90-100)	B+ (77-79)	C+ (65-69)	D (50-54)
A (85-89)	B (73-76)	C (60-64)	F (0-49)
A- (80-84)	B- (70-72)	C- (55-59)	

Course Policies on Missed or Late Academic Requirements

- Any excuse must be valid as per the current Dalhousie University guidelines. Given the fluidity of the situation, particularly in light of the COVID-19 pandemic, the university may change these policies and therefore certain leniencies will be in place. Medical notes are not absolutely required for HW extension requests. See more below.
- Students are expected to use the Student Declaration of Absence form for homework extension requests, and can be used as many times as needed.
 See more below.
- If a student misses a deadline, arrangements can be made (following one-on-one discussions) for later submissions or substitutions. See more below.
- Regarding missed HWs, the instructor must be contacted and a one-on-one discussion with a legitimate excuse, as per Dalhousie University guidelines.



- On missed midterm: Student's final grade will now constitute a 50% weight on their final exam.

Course Policies related to Academic Integrity

Students are expected to work on their own, consult regularly with the instructor as they progress through the exercises, and submit individual homework assignments. If students do work together on assignments, they must include the names of the other students with whom they collaborated on their submitted material. If you use ChatGPT or another type of AI, please indicate this on your submitted assignments.

Learning Objectives

The objective of this course is for the student to develop a solid understanding of Lebesgue's theory of integration including the fundamental theorems. At this level, it is expected that the student is adept at creating and writing mathematical arguments, but this course will refine those skills. Specific outcomes for the student are:

- Refined skill in creating clear and correct mathematics.
- Familiarity with the theory of measure and integration as presented in the course.
- The ability to use concepts from the course to analyse novel problems appropriate to the level of the course.
- A demonstrable overview of how the major theorems presented in the course depend on the web of preliminary results.



Course Content

Lecture topics (some of these topics will require more than one lecture):

- Preliminaries in topology
- Sigma-algebras
- Definition of measures
- Construction of measures: Outer measure and Lebesgue's construction
- Nonmeasurable sets
- Measurable functions
- Lusin's theorem
- Lebesgue integral
- Monotone convergence theorem
- Fatou's lemma
- Dominated convergence theorem
- Properties of the Lebesgue integral
- Product measures
- Fubini's theorem
- Radon-Nikodym theorem (if time permits)
- Lp-spaces: Completeness and duality (if time permits)



University Policies and Statements

Recognition of Mi'kmaq Territory

Dalhousie University would like to acknowledge that the University is on Traditional Mi'kmaq Territory. The Elders in Residence program provides students with access to First Nations elders for guidance, counsel, and support. Visit or e-mail the Indigenous Student Centre at 1321 Edward St or elders@dal.ca. Additional information regarding the Indigenous Student Centre can be found at: https://www.dal.ca/campus_life/communities/indigenous.html

Internationalization

At Dalhousie, 'thinking and acting globally' enhances the quality and impact of education, supporting learning that is "interdisciplinary, cross-cultural, global in reach, and orientated toward solving problems that extend across national borders." Additional internationalization information can be found at: https://www.dal.ca/about-dal/internationalization.html

Academic Integrity

At Dalhousie University, we are guided in all our work by the values of academic integrity: honesty, trust, fairness, responsibility, and respect. As a student, you are required to demonstrate these values in all the work you do. The University provides policies and procedures that every member of the university community is required to follow to ensure academic integrity. Additional academic integrity information can be found at:

https://www.dal.ca/dept/university_secretariat/academic-integrity.html

Accessibility

The Student Accessibility Centre is Dalhousie's centre of expertise for matters related to student accessibility and accommodation. If there are aspects of the design, instruction, and/or experiences within this course (online or in-person) that result in barriers to your inclusion, please contact the Student Accessibility Centre (https://www.dal.ca/campus_life/academic-support/accessibility.html) for all courses offered by Dalhousie with the exception of Truro. For courses offered by the



Faculty of Agriculture, please contact the Student Success Centre in Truro (https://www.dal.ca/about-dal/agricultural-campus/student-success-centre.html)

Conduct in the Classroom – Culture of Respect

Substantial and constructive dialogue on challenging issues is an important part of academic inquiry and exchange. It requires willingness to listen and tolerance of opposing points of view. Consideration of individual differences and alternative viewpoints is required of all class members, towards each other, towards instructors, and towards guest speakers. While expressions of differing perspectives are welcome and encouraged, the words and language used should remain within acceptable bounds of civility and respect.

Diversity and Inclusion – Culture of Respect

Every person at Dalhousie has a right to be respected and safe. We believe inclusiveness is fundamental to education. We stand for equality. Dalhousie is strengthened in our diversity. We are a respectful and inclusive community. We are committed to being a place where everyone feels welcome and supported, which is why our Strategic Direction prioritizes fostering a culture of diversity and inclusiveness (Strategic Priority 5.2). Additional diversity and inclusion information can be found at: http://www.dal.ca/cultureofrespect.html

Student Code of Conduct

Everyone at Dalhousie is expected to treat others with dignity and respect. The Code of Student Conduct allows Dalhousie to take disciplinary action if students don't follow this community expectation. When appropriate, violations of the code can be resolved in a reasonable and informal manner - perhaps through a restorative justice process. If an informal resolution can't be reached, or would be inappropriate, procedures exist for formal dispute resolution. The full Code of Student Conduct can be found at:

https://www.dal.ca/dept/university_secretariat/policies/student-life/code-of-student-conduct.html

Fair Dealing Policy

The Dalhousie University Fair Dealing Policy provides guidance for the limited use of copyright protected material without the risk of infringement and without having to



seek the permission of copyright owners. It is intended to provide a balance between the rights of creators and the rights of users at Dalhousie. Additional information regarding the Fair Dealing Policy can be found at: https://www.dal.ca/dept/university_secretariat/policies/academic/fair-dealing-policy-.html

Originality Checking Software

The course instructor may use Dalhousie's approved originality checking software and Google to check the originality of any work submitted for credit, in accordance with the Student Submission of Assignments and Use of Originality Checking Software Policy. Students are free, without penalty of grade, to choose an alternative method of attesting to the authenticity of their work and must inform the instructor no later than the last day to add/drop classes of their intent to choose an alternate method. Additional information regarding Originality Checking Software can be found at:

https://www.dal.ca/dept/university_secretariat/policies/academic/student-submission-of-assignments-and-use-of-originality-checking-software-policy-.html

Student Use of Course Materials

Course materials are designed for use as part of this course at Dalhousie University and are the property of the instructor unless otherwise stated. Third party copyrighted materials (such as books, journal articles, music, videos, etc.) have either been licensed for use in this course or fall under an exception or limitation in Canadian Copyright law. Copying this course material for distribution (e.g. uploading to a commercial third-party website) may lead to a violation of Copyright law.