Faculty of Science Course Syllabus (Section A)
Department of Mathematics and Statistics, Dalhousie University
Fall 2023

MATH 3790 - Mathematical Problem Solving

Dalhousie University acknowledges that we are in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People and pays respect to the Indigenous knowledges held by the Mi'kmaq People, and to the wisdom of their Elders past and present. The Mi'kmaq People signed Peace and Friendship

Treaties with the Crown, and section 35 of the Constitution Act, 1982 recognizes and affirms Aboriginal and Treaty rights. We are all Treaty people.

Dalhousie University also acknowledges the histories, contributions, and legacies of African Nova Scotians, who have been here for over 400 years

1 Instructor & Course Component Details

- **Instructor:** Dr. Dorette Pronk,   Office: Chase 302,   email: pronkd@dal.ca

- **Lectures:** MW 8:35 - 9:55 AM in Chase 319
  Lectures are in person and attendance is mandatory. However, if you are not able to attend due to illness, please contact me and I will run a livestream on the following Zoom link:

    `https://us02web.zoom.us/j/84727575330?pwd=ZGlJdmJWLzUvdnhKaFR4NUliVWd1Zz09`

  (or meeting ID: 847 2757 5330 with Password: Polya)

- **Office Hours:** M 10:30 - 11:30 AM, T 11 AM - 12 PM, 4 - 5 PM, or by appointment.
  Office hours at the scheduled times are normally in person in my office, but all office hour appointments can also be scheduled on Zoom. The link for office hours is:

    `https://us02web.zoom.us/j/87158814389?pwd=eUpmb01HQnVOYkEzeGVmZVNKUGVxdz09`

  (or meeting ID: 871 5881 4389 with Password: Problems)

- **Math Club Sessions:** the high school club meets on Tuesdays from 6 till 8 PM in room 319; the time for the junior club (for middle school and advanced elementary school students) is on Wednesdays from 5 till 6 PM, also in room 319.

2 General Information

Course Description: This class will provide an introduction to techniques for solving mathematical problems of the sort encountered in competitions (such as the Mathematical Olympiad or the Putnam competition). There will be self-contained modules developing techniques from several branches of mathematics including number theory, combinatorics, geometry and analysis. The majority of the class time, however, will be devoted to examining examples. Students will be expected to prepare and present in class solutions to assigned problems.

Prerequisites: MATH 1000 and MATH 1010 or equivalent, or consent of the instructor.

Corequisites: MATH 1030 or MATH 2030

2.1 Learning Objectives
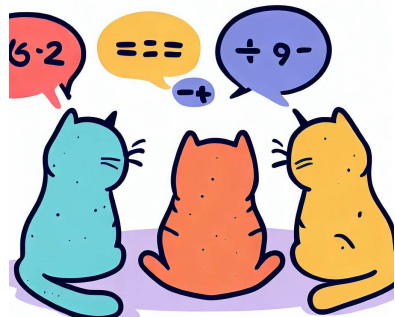
- Learn techniques for approaching competition problems as well as real life modeling problems and mathematical research problems.
- Learn to work effectively in small groups.
- Coach (pre-)university students in mathematical problem solving.
- Learn to communicate mathematical results with a variety of audiences.
- Analyse solutions by others and learn to spot flaws or gaps in their arguments.
- Learn techniques for tightening mathematical arguments.

Class Structure: Generally a lecture will start with a discussion by the students on the progress they have made on the "in-class problems" given at the end of the previous lecture and an effort to complete the solutions. Then I will introduce the next topic and some of the mathematical results you may want to use and give you new problems. You will have then some time to work in small groups on the problems.
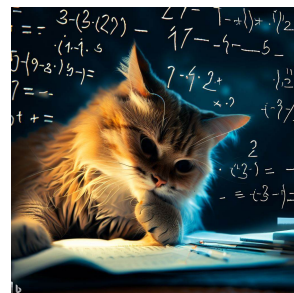
Textbooks: The following three textbooks will be used as sources for topics and problems. All are freely available online.

- Paul Zeitz, *The Art and Craft of Problem Solving*, Second Edition, John Wiley and Sons, 2006. https://kheavan.files.wordpress.com/2010/06/paul-zeitz-author-the-art-and-craft-of-problem-solving-2edwiley2006(pdf

- Răzvan Gelca, Titu Andreescu, *Putnam and Beyond*, Springer Verlag, 2007. https://mathematicalolympiads.files.wordpress.com/2012/08/putnam-and-beyond.pdf

- Loren C. Larson, *Problem-Solving Through Problems*, Springer Verlag, 1983. https://web02.gonzaga.edu/faculty/axon/Larson_problem-solving.pdf
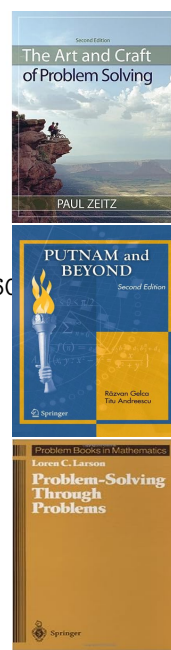
3   Course Assessment

Homework: This class has two types of regularly scheduled homework:

- The "in-class problems" that get assigned at the end of each class and need your attention with an attempt to solve them before the next class. You should make sure that you have made progress on at least one of them.

- The weekly home-work that is due each Thursday in the Brightspace dropbox. Please submit your work in one pdf file. This homework will have three sections: othe first section asks you to write up a nice solution for one of the in-class problems, the second section has problems that are very similar to the ones we covered in class, and the third section has a new challenge that can be solved with the techniques discussed but it may not be obvious how. For the last section I am mostly looking for good attempts. Generally a complete solution in this section will result in bonus points on the assignment.
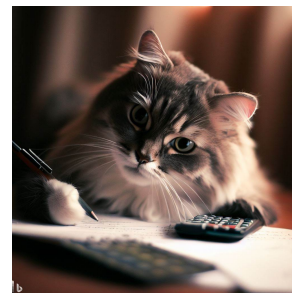
Math Club: There will be two math clubs this fall: the junior and the senior level club.The students come to the clubs because they like math competition problems and the creativity that is required to solve them. You are asked to sign up to attend the club four times: twice as an observer and TA and twice as a presenter. You should observe at least once before presenting. I will post the signup sheet in Brightspace and also bring it to class. First come, first served, so don't wait!
Your duties for the days you sign up for are as follows:

- If you are an *observer*, you may want to touch base with the presenter before club to find out what they are teaching on, but this is not rerquired. You are required to write a report about your experience that includes answers to the questions in the file called "reflection observer" posted in Brightspace. This report is due a week after you observe.

- If you are a *presenter*, you need to meet with me at least a week before the math club to decide on the topic you want to teach on (it is better to come earlier and discuss some general ideas before you make your final decision). At least three days before the actual club day you need to send me your draft hand-out for the class. I will then send it back with suggestions for editing and perhaps additional problems. You need to have the final version of you hand-out ready the day before you teach so that I can make additional copies for you. On the hand-out leave enough white space for the students in the club to work on. After you have taught, you need to write a presenters report that includes answers to the questions in the file called "reflection presenter" posted on Brightspace. This report is due one week after you teach.

Midterm Test: The midterm test is on Wednesday, October 25, in class.

Final Exam: The final exam is 3 hours long. The date and time for the final exam is set by the registrar during the official Dalhousie exam period from Dec. 8 until Dec. 18, 2023. If you plan to depart from campus at the end of the semester, please make your plans after the registrar has announced the exam schedule, or plan to leave after Dec. 18. Unfortunately, there are no opportunities to write the exam early or remotely.

Course score: I will use the maximum of the following two columns for you:

Weekly Assignments	25%	Weekly Assignments	25%
Class Participation	10%	Class Participation	10%
Math Club Presentations	10%	Math Club Presentations	10%
Math Club Reflections	10%	Math Club Reflections	10%
Midterm Test	20%		
Final Exam	25%	Final Exam	45%

The grading scheme for this course will follow the standard scale set by Dalhousie University.
`https://www.dal.ca/campus_life/academic-support/grades-and-student-records/`
`grade-scale-and-definitions.html`

4 Student Accommodations

If there are aspects of the design, instruction, and/or experiences within this course that result in barriers to your inclusion please contact the Student Accessibility Centre `dal.ca/access`.

5 Welcome to new adventures!

The explorer is the person who is lost.
  –Tim Cahill, *Jaguars Ripped My Flesh*

6 Course topics and approximate schedule

In the table below I will abbreviate our books as: AaC for "Art and Craft of Problem Solving"; PaB for "Putnam and Beyond" and PtP for "Problem-Solving through Problems". This is an extremely tentative schedule. If there are topics you would like me to include, please tell me!

week 1 Exploration Part 1: get your bearings - heuristics and ways to approach problems.
(Sources: AaC Chapters 1 and 2, PtP Chapter 1.)

week 2 More heuristics and Exploration Part 2: verify/check your plans - principles for proving results.
(Sources: AaC Chapter 3, PaB Chapter 1, PtP Chapter 2.)

week 3 Further checking your plans: combining techniques - cross-over tactics in proofs.
(Source: AaC Chapter 4.)

week 4 Terrain Exploration Part 1: Polynomials
(Sources: AaC Chapter 5.2 and 5.4, PaB Chapter 2.2, PtP Chapter 4.)

week 5 Terrain Exploration Part 2: Linear Algebra
(Sources: AaC Chapter 5.2 and 5.4, PaB Chapter 2.3.)

week 6 Terrain Exploration Part 2: Linear Algebra continued

week 7 Terrain Exploration Part 3: Geometry

week 8 Terrain Exploration Part 4: Real Analysis

week 9 Terrain Exploration Part 4: Real Analysis continued

week 10 Terrain Exploration Part 5: Number Theory

week 11 Terrain Exploration Part 5: Number Theory continued

week 12 Terrain Exploration Part 6: Combinatorics

week 13 Terrain Exploration Part 6: Combinatorics continued and a review of our adventures.

7   Course Policies related to Academic Integrity

Submitting your written homework: You are strongly encouraged to collaborate with other students when working on homework and planning the journey to a solution. However, when you write up your solutions,you need to do so by yourself - I am not expecting to see the exact same solution more than once.

8   Course Policies on missed or late academic requirements

In the event that you are absent for three days or fewer resulting in missed or late academic requirements, you will be required to submit a Student Declaration of Absence Form to your instructor, see:
`https://www.dal.ca/campus_life/safety-respect/student-rights-and-responsibilities/academic-policies/student-absence.html`

I understand that circumstances can arise that can interfere with completing your work. I will drop your two lowest homework scores to function as a buffer for all.

If you are not able to teach math club due to illness, there are two options: if you are up for it, I may be able to run the session in hybrid format, or we need to find a replacement for you. So if you are coming down with anything please inform me as soon as possible, so that I may be able to find somebody to take your place. This is also a good reason to start preparing more than one week in advance!

Midterm and Final exams: In the event that you are unable to attend the midterm or final exam, please notify the instructor via email in advance to determine what alternatives may be possible.

9 Excursions: the Science Atlantic and Putnam Competitions

Although these are not course requirements, I strongly encourage you to register for both of these! They will give you a chance to put into practice what you have learned and meet with other explorers.

Science Atlantic This competition is part of the Science Atlantic Undergraduate Conference in Math, Stats and CS, which will be held at UPEI on October 13 and 14. The competition is on Friday. This is a team competition: you may participate in teams of two students. If you want to participate, send me an email. There will be funding from Dalhousie to cover your cost, but we need to apply together. We will be leaving early Friday morning and returning Saturday evening or Sunday morning.

Putnam The Putnam competition is on December 2, all day and you can write it here in the department. This is an individual competition. You need to register with me.
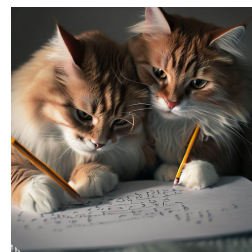
Note: most of the images in this syllabus were created with the chatgpt image generator.