# I, Mathematician

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What is mathematics? What makes an individual a mathematician? How does the rest of the world view mathematicians and their work? Despite the inherent difficulty in answering these questions, an attempt must be made for at least two reasons. Most branches of science are being studied mathematically these days, the way physics was in earlier times, and new ideas and new ways of thinking are cropping up all the time; it is essential to sift the grain from the chaff, mathematically speaking, for it could potentially lead to breakthroughs. Another reason is that public understanding and appreciation, and the concomitant support, are vital for the survival and growth of mathematics.

The difficulty alluded to above naturally leads to the existence of a wide variety of opinions, as expressed, for example, by the twenty-four mathematicians and the wife of one of them, a well-known group theorist, who contribute articles to the book under review. Almost all of them have something interesting to say, but some stand out. Before discussing the individual contributions, let's first look at the way the book is structured. There are plenary articles and secondary ones, with the former being subdivided into two groups, the first dealing with the question "Who Are Mathematicians?" although many of them stray far beyond that, while the second goes under the rubric "On Becoming a Mathematician." Secondary articles answer the question "Why I Became a Mathematician." There is a plenary article by Alan Schoenfeld although his name doesn't appear in the list of plenary contributors mentioned in the Preface, whereas Tom Apostol, James Milgram, and Robert Strichartz *are* mentioned as plenary contributors, even though there are no articles by them. More striking, however, is the absence, not only of dyed-in-the-wool applied mathematicians such as Joe Keller, but even of stalwarts like Peter Lax, who are known for their work in both pure and applied mathematics. Equally surprising is the absence of probabilists and mathematical statisticians. True, V.S. Varadarajan was a distinguished probabilist at the beginning of his career, but he later switched to representation theory. (Readers might like to know that no less a person than David Mumford said, in a lecture in 2000, one of the millennial ones, that we are now in "The Age of Stochasticity" and that perhaps the axioms of geometry should be stochasticized. By the way, he himself switched to mathematical problems of computer vision and left Harvard for Brown.) Finally, of the twenty-four mathematicians, four are women, two of whom contribute plenary articles.

Not all the essays in the book answer the questions mentioned at the beginning, or at least not the way one would expect. Some of them deal almost exclusively with teaching and math education; some are substantially autobiographical, and at least one, that by Manin, deals only with mathematics per se and its connection to art. Although there are all these differences, there is one invariant without question: the love of mathematics. Indeed, it permeates the whole book.

Since the number of contributors is large, we'll dispense with quotation marks when discussing a contribution; instead, we'll specify the contributor's name and unless otherwise stated, what follows will be either a summary of the author's contribution or a verbatim reproduction of it. The reviewer' comments will be either explicitly identified as such or enclosed in parentheses.

The piece by Atiyah is perhaps the shortest but packed with substance, so let's start with that. He remarks that mathematicians are thinkers and, in theory, could work on a desert island or even in a prisoner-of-war camp. (There are known examples of the latter.) This self-sufficiency causes the general public to view mathematicians with awe and incomprehension. (It becomes even more striking in the case of handicapped mathematicians – witness Pontryagin (blind) and Hawking (ALS).) Isolation and introversion can foster the creative imagination, but when pushed to extremes, can lead to mental illness. Mathematics shares the freedom of creative thought with art but is close to natural sciences and gets anchored to the physical world. Mathematics, because it is part of science, is hierarchical in the sense that all discoveries in it build on earlier work and aim at a unification of knowledge. Thus, for example, Hilbert could recognize advances made in the twenty-first century, but it is not clear if Bach would appreciate modern music. Science is collaborative, while art in general is not: mathematicians interested in the same topic or problems may collaborate, but it is hard to imagine two or more people composing a poem or a symphony. Although mathematicians can differ from one another in the same way as people in general do, what unites them is a common passion for mathematics and the use of rational methods, with understanding as the final goal.

Let's now turn to the essays by the duo Michael and Pam Ashbacher. Michael mentions three characteristics of mathematicians: quick learners, better able than most to cope with novelty; capable of intense concentration over extended periods of time; and some use intuition more than others. Mathematics consists in discovering patterns or structure in complex and apparently chaotic situations; some can use insights won by intuition to build a useful edifice using the tools of math: clarity, precision, abstraction, and rigor. He also gives a clear picture of the evolution of a mathematician through the various stages: undergraduate, graduate student, postdoctoral fellow, junior faculty member, and finally a full-fledged professor and active mathematician and part of a community working in a specific area. Pam Ashbacher's article is full of examples of mathematicians' other worldliness, absent-mindedness, and idiosyncrasies including the use of mathematical jargon to make sense of everyday life. She explains the adjustments spouses have to make and the joys of living with people absorbed in high-level thinking without much regard for material rewards: solving a difficult problem by creating tools that become useful in other investigations is the ultimate reward.

Let's turn our focus now to articles dealing primarily with teaching. Schoenfeld started out as a mathematician, but got interested in the techniques of problem-solving, which in turn led him to math education. (One reason he gives for the switch, the low probability of his doing groundbreaking work in math, isn't convincing. For one thing, rare events happen all the time – think of disasters or winning a lottery – and for another, his criterion would imply that most mathematicians should do something else. A more convincing argument proffered is that math education is a young field and affords opportunities for making significant contributions.) He loves it, and would like to entice others to the field. He says educational research is as exciting and challenging as research in math. Among the results of his research are new ways of teaching the meaning of slope and the finding that children's arithmetical errors are not random but systematic; he cites the work of people who designed a test that predicts, about half the time, the incorrect answers that students would get to subtraction problems. He says that before a child learns the correct way of doing, say subtraction, certain habits learned earlier must be unlearnt. He thinks of mathematics as sense-making and remarks on how rewarding it is to help children do that, and emphasizes the need for humility on the part of mathematicians who reach out.

Two things stand out in Hyman Bass's essay: one is the nature and importance of proofs – how a proof doesn't just establish truth but explains it. He says that proof analysis may reveal hypotheses weakly or not at all used and that a thorough analysis of a proof might lead to stronger conclusions than stated in the theorem, or the dropping of some hypotheses. The other is that in teaching elementary and secondary school students, it is better to ask questions rather than provide answers, thereby inviting the students to think on their own and also interact with other students and not just with the teacher. He illustrates this with a diagram, "Instructional Triangle," where one vertex represents teachers, another content, and the third students interacting with other students. Finally, he gives a beautiful example in which third grade students discuss and come up with a way of proving that the sum of two odd numbers is even.

Sol Garfunkel started out as a logician and taught at Cornell for three years. He was interested in both research and teaching, but following the advice of the Chair there as well as that of John Gardner, then Secretary of Health, Education, and Welfare, he did both for eleven years at UCONN, all the while leaning more and more toward teaching. He got excited by an education project at MIT, where techniques of teaching calculus using a variety of experiments that students could do at home were being developed. He got hooked and became a math educator. He describes in some detail the differences between mathematicians and math educators. He came to believe it's a positively good thing to teach how mathematics is used. Along with some colleagues he founded COMAP (Consortium for Mathematics and Its Applications), which produces material – modules, books, TV series – that help explain applications of mathematics including newer ones as they come along. It also conducts two annual contests: MCM (The Mathematical contest in Modeling) and ICM (The Inter-Disciplinary Contest in Modeling). All in all, Garfunkel and his colleagues have been performing yeomen's service in the cause of supporting and strengthening an integrated view of mathematics and of narrowing the chasm between pure and applied mathematics. To the extent that he began as a pure mathematician and ended up as an applied one, he resembles Mumford.

Ian Stewart stopped teaching undergraduate classes some time ago, splitting his time between research and efforts to improve the public understanding of science, the latter done primarily through newspaper and magazine articles, making podcasts and contributing to websites, among other things. After Martin Gardner's retirement, he wrote a column for Scientific American on mathematical games and puzzles for about ten years. He believes, based on experience and observation, that there is such a thing as mathematical talent, contrary to popular claims that everybody has the same natural abilities and that the only reason for differences in achievement is hard work. Like V.S. Varadarajan, whose essay follows his, he says there are dual demands on mathematics, internal and external. Internal ones could be logical reasoning, precision, and consistency, while external ones could be solving problems arising in science and engineering. He says that "pure" and "applied" are not opposite sides of a coin, but rather aspects that complement each other. (One might say that applied math supplies problems to pure math – think of Stokes theorem or the ergodic hypothesis.) For instance, a mathematician may be interested in proving the existence, uniqueness, and regularity of solutions of an IVP for a DE and at the same time try to answer the question asking, a la Mark Kac, if one can hear the shape of a drum. One characteristic of mathematicians he emphasizes is the utter lack of fear of anything technical. Finally, he says that even though schisms exist in mathematics – pure vs. applied, constructivist vs. non-constructivist – the best thing is to live and let live and not disparage people on the other side of the tracks.

Varadarajan begins his essay with a reference to C.P. Snow's book "The Two Cultures" and wonders if more than one in ten of highly educated people would be able to answer the question "What do you mean by mass or acceleration?" (To be fair, mass is a rather difficult concept, especially if one takes relativity into account.) Like Stewart, he speaks of the dual nature of mathematics, the inner and the outer, but adds to the inner part the qualities of beauty, symmetry, and the existence of connections among different parts of the subject. He says that nevertheless, these models of internally created structures are the ones that mimic most accurately the models occurring in nature. In other words, work done for purely internal reasons often turns out to be useful in describing the phenomenological world. Why this happens is a mystery.

He then goes on to illustrate this by discussing the use of infinite dimensional spaces (Hilbert spaces) to describe the states of quantum systems; probability, especially Wiener's work, in which probability is defined on the space of paths in Brownian motion, accounting for the fact that the paths are continuous but have no tangents anywhere; and geometry, where Einstein's theory of general relativity uses Riemannian geometry to describe space-time and gravity. He sees an urgent need for mathematicians to abandon their ivory towers and communicate the beauty, grandeur, and usefulness to lay people. This is important because the public needs to be convinced that fundamental research in mathematics and science is worth pursuing. Also, a well-informed public can make good decisions in matters where science and politics meet.

Yuri Manin starts his essay with Coco Chanel's explanation of why she liked the painting "Harlequin with Violin" by Picasso – it was "convincing," reminding her of a logarithm table. This quality of being convincing or persuasive is a theme that recurs throughout the essay, rather like a leitmotif.

He says that the Romans' lack of interest in Greek mathematics and science, as evidenced by their use of Roman numerals, which he regards as unimaginative, led to a millennium-long hiatus in the progress of mathematics in the West and concludes that the introduction of a place-value number system and the concept of zero were essential in the revival and growth of mathematics. Appealing to an interpretative tradition, he says that Archimedes did not regard his engineering endeavors as "applications" of his mathematics but rather as an unwanted distraction. (One wonders, though. Readers of Alfred Renyi's "Dialogs on Mathematics" certainly wouldn't get that impression, even though the dialogs, rather than being factual, were *imagined* by Renyi.)

Manin comments on how the positional decimal notation developed into the "unique universal language of modernity" and describes how its semantics, syntax, and pragmatics work. He then makes the interesting statement that Leibnitz's insight that not only numerical manipulations, but any rigorous, logical sequence of thoughts that derives conclusions from initial axioms can be reduced to computations presages the work of Godel and the other great logicians of the twentieth century. He then moves on to the dual ways of conveying mathematical meaning: words and formulas on one hand and diagrams on the other, giving both elementary examples and some in the areas of homotopy and category theory. He then reverts to the theme of the "persuasiveness" of mathematics, citing examples of how it can be achieved. He concludes with a return to the relation between art and mathematics, saying that there is a parallel between working out the step-by-step details of a proof to make it a convincing whole and a musician working out the details of small movements until they become automatic and can be synthesized, say, in Bach's sonata for solo violin.

T.W. Korner's essay offers advice on how to lecture. He says that preparation and the ability to think on one's feet are important. (Perhaps one could add a suggestion the reviewer once received from a visiting Australian mathematician: always empty your bladder before a lecture.)

Peter Casazza and Steven Krantz are somewhat pessimistic about the perception of mathematics and what mathematicians do not only by the lay public but also by administrators who are not mathematically literate. Casazza offers a survival guide, while Krantz describes the frustrating fights he has had with administrators in his quest for resources and support. He offers some tips on how things can be improved: it helps if there are analysts and geometers working on differential equations, probabilists and statisticians, etc. on the faculty. He speaks of the need for communicating mathematics to the lay public. (Some names immediately come to mind: Marc du Sautoy, Simon Singh, Ian Stewart, and perhaps Steven Strogatz.)

Underwood Dudley, convinced that mathematicians are different from the rest of mankind, presents many anecdotes and quotations – not too benign or favorable to mathematicians – in support of his view. The overall impression one gets from his essay is in stark contrast to what Ed Nelson said some time ago in the American Math Monthly about how mathematicians are surprisingly nice. Although he didn't offer an explanation, one could reasonably guess that the high status mathematicians accord to truth has some side-effects such as mathematicians admitting error and saying "sorry" and recognizing the priority of other people's work when that is called for. The same quality explains a senior mathematician bending over backwards to give credit to a younger colleague or a student for a good idea or technique. Comparing all this to what Dudley says, though, leads to the inescapable conclusion that mathematicians can indeed be very different from one another.

The late Jon Borwein, a former colleague of the reviewer, was a strong advocate, and practitioner, of experimentation in mathematics. He claimed that the diagrams obtained during experimentation and the clues they provide lead to inductive, non-traditional proofs. He was more sanguine than Casazza or Krantz about the place of mathematics and mathematicians in society. He argued that mathematics is more like poetry than any other form of art.

Roger Cooke's article is the only one in the collection which asks questions about the extent to which one's upbringing and social background influence one's desire to be a mathematician. In an attempt to answer them, he gives a remarkably honest account of the racism and sexism prevalent in his childhood and youth in the community in which he grew up. He wonders how well he would have done if he had had to compete with women and minorities on a level playing field. The question of the utility of abstract knowledge loomed large in his imagination. For instance, while a graduate student, he wondered how his study of Banach algebras helped the taxpayers who funded his education. He conducted a semi-scholarly survey of 85 prominent mathematicians and found a strong correlation between the profession of one's parents, usually the father, and the fact that one is a mathematician. He also believes that the social and ethical values instilled in one's childhood continue to influence a person who does become a mathematician.

Keith Devlin says that the two things that led him to become a mathematician were the bad way calculus was taught in high school in England when he was young and his realization of the limitations of human language. Initially, he was a Platonist and thought of mathematics as a science of patterns. He says mathematicians fall into the not mutually exclusive classes of problem solvers and theory builders (along the lines of Freeman Dyson's frogs and birds) and sees himself as a big-picture guy. Concerning Platonism and its opposite, he says that once the basic ideas and the foundations of an area are in place, one has the illusion that the few fundamental ideas that survive existed already. He remarks about the difficulty of defining what constitutes doing mathematics, for when one is aware of doing mathematics, one is no longer doing it. (In this respect, doing mathematics is like engaging in any creative activity: the moment one is aware of it, one is no longer engaged – a certain unselfconsciousness goes with it.) Perhaps a good way to describe it is to say that one inhabits a virtual world, one in which play with mental constructs takes place. Having started out working in set theory and foundations, he cautions against confusing models with the objects they model, such as Dedekind cuts (ordered pairs of sets of rationals) and real numbers. He thinks of mathematics as a body of knowledge that results from mathematical thinking, a difficult-to-define form of human activity. (Thinking mathematically about objects outside of mathematics can, contrary to what he says, lead to new mathematics. For example, the ergodic hypothesis put forward by Boltzmann in statistical mechanics – space averages and time averages are equal – is the origin of modern ergodic theory, the first important result in which was Birkhoff's pointwise ergodic theorem.)

Jane Hawkins starts her essay asking what a mathematician is, but after some remarks, abandons it because of the impossibility of the task. Being a woman, she is naturally interested in the stories of women who came before her and discusses some of them but surprisingly omits mention of Kovalevskaya. She talks about the relative importance of teaching and research and mentions computers and technology as important aids, particularly in her area of ergodic theory and dynamical systems. She discusses peculiarities and eccentricities of mathematicians and mentions some problems they are prone to, such as deep-vein thrombosis and divorce.

The first part of Harold R. Parks's essay explores the origins of abstract/mathematical thinking and concludes with an interesting quotation of the Greek historian Herodotus's statement that geometry arose out of the need to measure and estimate the size of agricultural plots in the Nile valley of Egypt, whence it moved to Greece. It was developed there in a systematic manner by the creation and use of the axiomatic and deductive method. The heart of the essay, however, is a valuable guide to prospective students of mathematics, all the way from elementary school to graduate school. Since interest in, and a talent for, mathematics first shows up in elementary or high school, he says that perhaps teachers, like physicians, should follow the precept "First, do no harm." Overall, he offers sound and helpful advice on dealing with practical problems, including teaching, that graduate students and postdoctoral fellows looking for jobs face.

Mei-Chi Shaw's article is really a very well-told story of a Chinese female mathematician, right from her childhood all the way to the time she becomes a well-established mathematician working at a prominent university in the US. (Strictly speaking, she is from Taiwan, but her parents fled there in 1949 from mainland China when the nationalist government lost the civil war to the communists.) It is full of interesting details including those concerning the excellent education she received as an undergraduate at National Taiwan University and as a graduate student at Princeton. All in all, it is an inspiring tale that could serve as a helpful guide to ambitious prospective students, especially females from outside North America.

The remaining six articles, whose theme is "Why I Became A Mathematician," differ from one another because of differences in individual circumstances, but all the authors have in common a passion for mathematics. Each story is interesting in its own right, but the reviewer particularly likes those of Jenny Harrison, the only contributor who says she does yoga and meditation, and of Rodolfo Torres, who is originally from Argentina and is the only Spanish-speaking contributor. Unfortunately, as far as the reviewer can tell, there are no black mathematicians among the twenty-five authors.