IMHOTEP'S

LEGACY ACADEMY

COMPLEX GEOMETRY: EXPLORING A 3-D 12-POINTED STAR

Grade 6 Activity Plan

Reviews and Updates

6.8 Complex Geometry: Exploring a 3-D 12-Pointed Star

Objectives:
1. To learn how to construct a complicated shape using many simple shapes.
2. To understand the construction of a dodecahedron and build one using paper and tape.
3. To learn the concept of **tessellations** of three-dimensional space, and investigate whether a dodecahedron will tessellate three-dimensional space.

Keywords/concepts: Dimension, Pyramid, Tri-Pyramid, Dodecahedron, Tessellate, Stellation, Rhombus, Platonic Solids, Space.

Curriculum outcomes:
Grade 6: G02.06, G04.03.
Grade 7: E1, E4, E5, E6.
Grade 8: E6, E7.
Grade 9: E4, E5.

Take-home product: Paper model of a stellated rhombic dodecahedron.

Written by: Dario Brooks, July 2016.
Edited by: Bai Bintou Kaira, July 2016.

Segment	Details
African Proverb and Cultural Relevance (5 min.)	"Wisdom is like a baobab tree; no one individual can embrace it." **Ewe proverb.**
Pre-test (10 min.)	Ask students if they know the meaning of the "dimension" of a shape or space. Brainstorm some 2-D shapes and their 3-D counterparts (e.g. 2-D: square, 3-D: cube).
Background (15 min.)	Discuss the concept of two- and –three-dimensional space if students are not already familiar. Explain how we can use a 2-D object to create a 3-D object by folding. Introduce the *dodecahedron* and the other Platonic solids.
Activity 1 (45 min.)	Construct a 3-D stellated rhombic dodecahedron by building pyramids out of paper and taping them together.
Post-test (15 min.)	Geometry Scavenger Hunt

Suggested interpretation of the proverb: Wisdom is something that is to be shared by many people, not just one individual. When learning, it takes teamwork, cooperation, and patience to truly learn the object of study. After one has gained knowledge through learning, one should share their new found knowledge with others around them so they too may benefit from the wealth of wisdom.

BACKGROUND INFORMATION

Geometry is everywhere. Literally everything in the universe has a geometric structure – a pencil, a table, a dog, humans, even empty space itself – and so can be described using geometry. Geometry is the oldest branch of mathematics: humans in ancient civilizations would observe the world around them, ask questions about why things look the way they do, and would discuss methods to describe these things. Since geometry began as the study of natural surroundings, it has been deeply developed over many years based on descriptions of the world around us. Professional **geometers** – people who study geometry and use it to solve practical or abstract problems – often can be seen working alongside physicists and chemists. Geometers can also be found working on problems in physics and chemistry themselves, or elsewhere, because geometry has applications to many different fields in science and technology.

Geometry that is most familiar to people today exists in two or three dimensions. **Two-dimensional (2-D)** geometry is anything that takes place in flat space, with width and height; for example, anything someone draws on paper is a two-dimensional object. **Three-dimensional (3-D)** geometry is the study of things with width, height, and depth. Besides paper thin objects, everything one will encounter in everyday life is three-dimensional, because the universe itself is 3-D* (in fact even paper is three-dimensional because it has thickness, but its thickness is so small that we consider paper and the drawings on it to be close enough to two-dimensional). It is also possible to turn a 2-D object into a 3-D object. For example, if that piece of paper with the drawing on it is folded, a two-dimensional object turns into a three-dimensional object. This concept will be explored more in the following activity.

Cubes, cylinders, rectangular prisms, and spheres are some common three-dimensional objects. There are five special types of 3-D objects, called the **Platonic Solids**, named after the ancient Greek philosopher Plato. Plato believed that the five "classical elements" – earth, air, water, fire, and aether (similar to air, filling the space above the Earth's atmosphere) – were made of these solids. The five Platonic solids are the **tetrahedron** (triangular pyramid) with four faces, **hexahedron** (cube) with six faces, **octahedron** with eight faces, **dodecahedron** with 12 faces, and **icosahedron** with 20 faces. Plato believed that the universe itself at its most basic core was built out of dodecahedra. The famous mathematician and philosopher Bertrand Russell wrote a short story in 1954 that suggests he believed the universe is one giant dodecahedron as well; so it is appropriate that the dodecahedron is the solid chosen to investigate in this activity. Participants will utilize the ability of building three-dimensional shapes by folding paper to construct their own star-shaped dodecahedron, called a **stellated rhombic dodecahedron**.

* If students are curious about the universe, explain that the universe is in fact four-dimensional, with the fourth dimension being time.

Activity 1: Building a Three-Dimensional 12-Pointed Star

Purpose: To explore the properties of simple geometric figures and a complex geometric figure through construction, and to relate these properties to the concept of three-dimensional space.

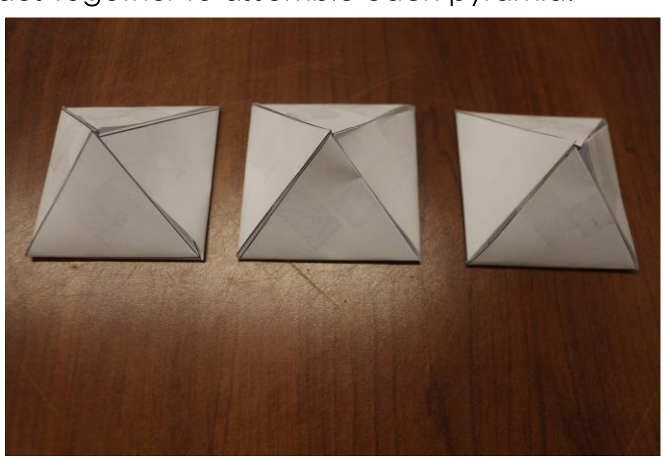
Suggested format: Students work independently at first, then in groups at the end.

Item	**Quantity (10 students)**
Paper printouts of pyramid pattern	60 (24 patterns per student pair, two patterns per sheet)
Scissors	10
Transparent tape	10

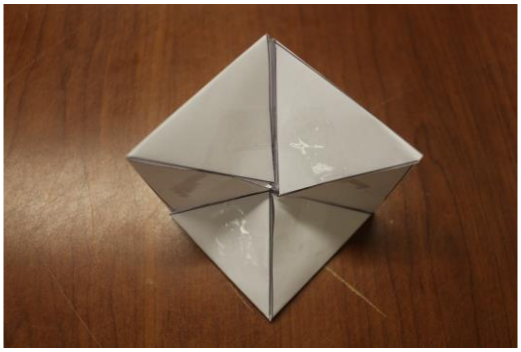
TO BE DONE BY MENTORS (prior to activity): Print off sheets of pyramid patterns, with two patterns per letter-sized sheet of paper, as on pages 5 and 6 of pdf in https://www.exploratorium.edu/geometryplayground/Activities/GP_Activities_6-8/Exploring%20a%20Complex%20Space-Filling%20Shape_6-8_v5.pdf.

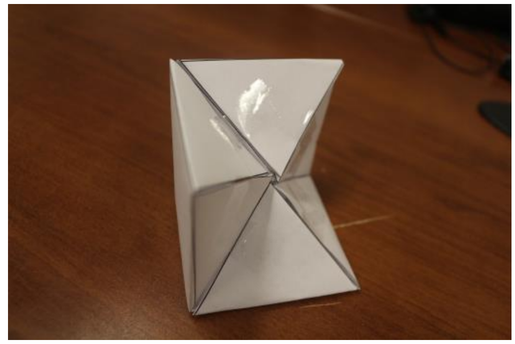
Procedure:

1. **Mentors:** Pair up students.
2. Cut out three pyramid patterns.
   Tip: To save a lot of time, cut many patterns out at once by placing multiple sheets on top of each other and cutting along the lines. One can easily cut out four patterns at once.
3. Fold each pyramid pattern along the lines. Each fold should be away from you (a "mountain fold").
   Tip: If one is careful, it is possible to save more time by folding three pyramid patterns at the same time by laying them on top of each other and folding.
4. Tape the sides together to assemble each pyramid.

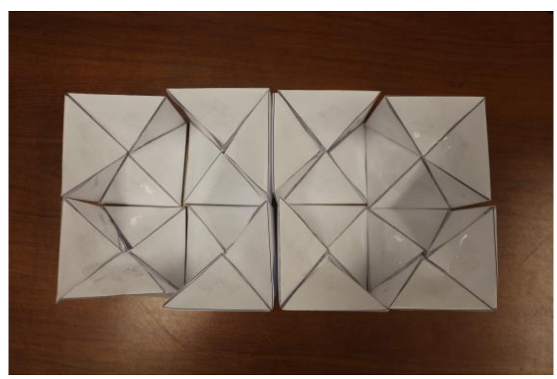


5. Tape the three pyramids together as shown below. This new shape will be called a “**tri-pyramid**”.

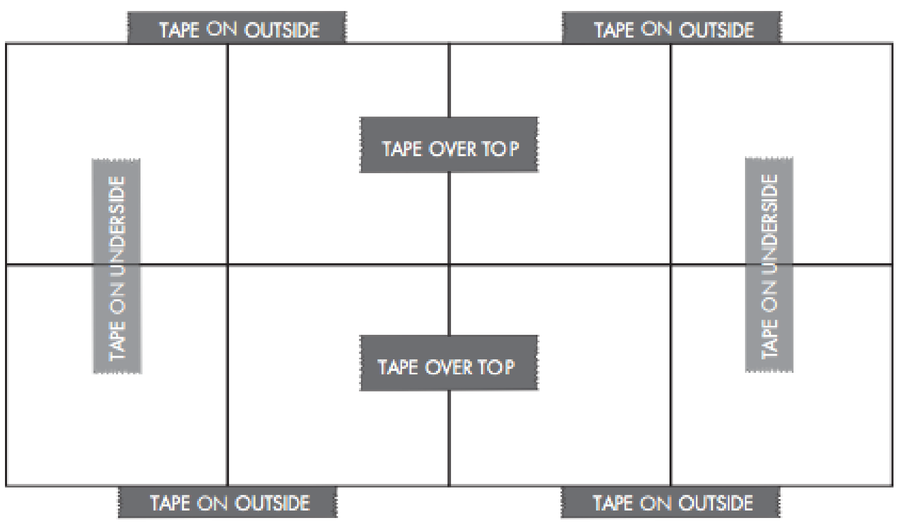




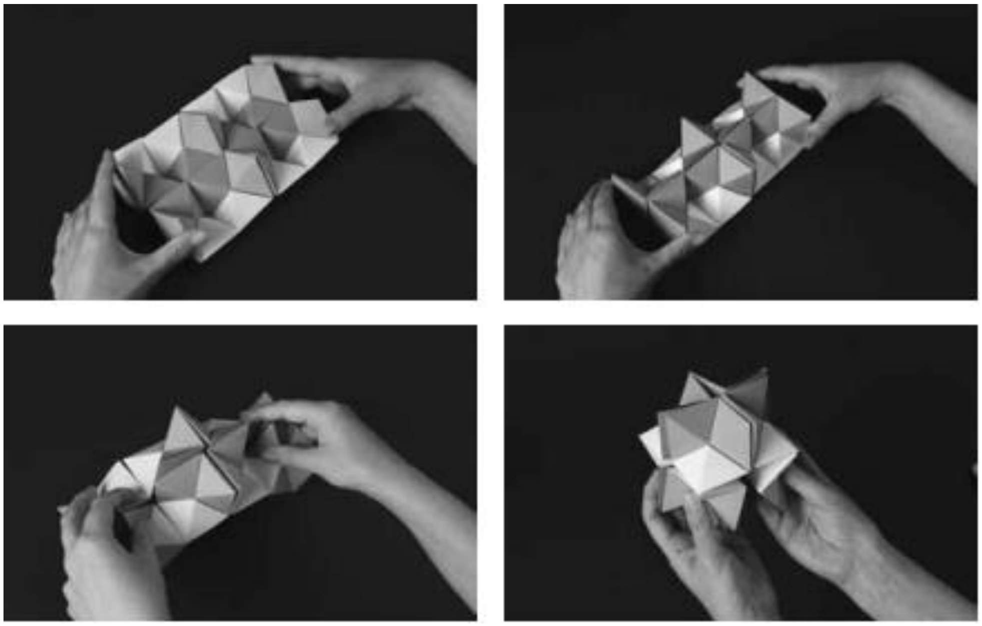
6. Repeat steps 1-4 to create seven more tri-pyramids, for a total of eight. Arrange the eight tri-pyramids to form two “cups” side-by-side, as in the pictures below.

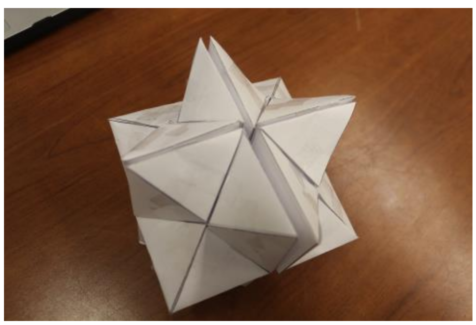


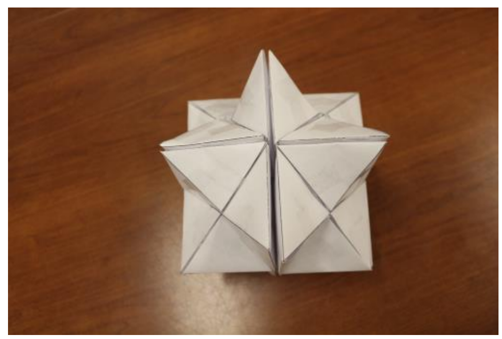
7. Tape the eight tri-pyramids together as shown in the diagram below. Each square in the diagram represents one tri-pyramid, from a bird's-eye view.



8. Manipulate the taped model as in the pictures below to form a 12-pointed star.







9. Colour the sides of the star with colours of your choice.

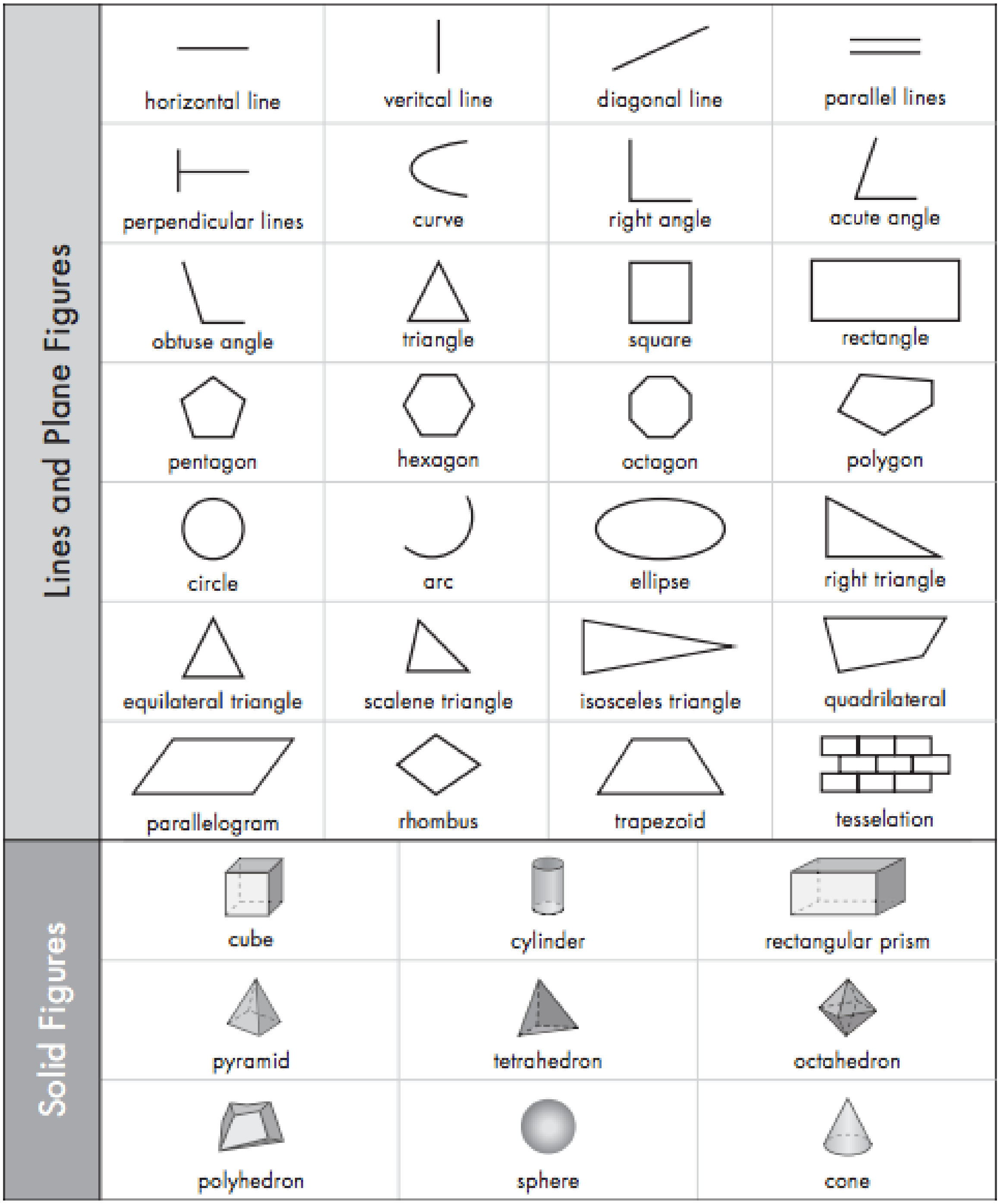
Note: The spiky shapes sticking out of the star are called **stellations**. The base of each stellation is a diamond shape, called a **rhombus**. 12 stellations in total can be found on the star. In geometry, a solid figure with 12 faces is called a **dodecahedron.** The 12-pointed star we have constructed is therefore known as a **stellated rhombic dodecahedron**.

Optional exploration: We know that cubes can fill any region of 3-D space with no gaps between them – this is known as *tessellation of 3-D space*, and so cubes are said to **tessellate** three-dimensional space. Discuss whether stellated rhombic dodecahedrons will tessellate three-dimensional space. Try to fit them together with no gaps in between in pairs or as a group.

Post Test:

Geometry Scavenger Hunt:

Explore the surroundings (classroom, playground, etc.) to find interesting geometric figures. Use the geometric shapes chart below to identify familiar shapes you find. See how many of them you can find in your surroundings. For bonus points, draw the objects you find that resemble the figures in the chart.


Lines and Plane Figures
horizontal line
veritcal line
diagonal line
parallel lines
perpendicular lines
curve
right angle
acute angle
obtuse angle
triangle
square
rectangle
pentagon
hexagon
octagon
polygon
circle
arc
ellipse
right triangle
equilateral triangle
scalene triangle
isosceles triangle
quadrilateral
parallelogram
rhombus
trapezoid
tesselation
Solid Figures
cube
cylinder
rectangular prism
pyramid
tetrahedron
octahedron
polyhedron
sphere
cone

References and additional resources:

Background information:
http://web.archive.org/web/20120425223703/http://physicsworld.com/cws/article/news/2003/oct/08/is-the-universe-a-dodecahedron
https://arxiv.org/abs/astro-ph/0310253
https://www.encyclopediaofmath.org/index.php/Platonic_solids

Activity 1:
https://www.exploratorium.edu/geometryplayground/Activities/GP_Activities_6-8/Exploring%20a%20Complex%20Space-Filling%20Shape_6-8_v5.pdf

Post-Test:
https://www.exploratorium.edu/geometryplayground/Activities/GP_OutdoorActivities/GeometryScavengerHunt.pdf

Many additional activities:
https://www.exploratorium.edu/geometryplayground/resources.php

Materials List

Item	Store	Cost
Westcott Kleen Earth 8" Scissors; 3 pack	Wal-Mart	$9.97
Scotch Magic Tape, 6 pack	Wal-Mart	$10.67
Crayola Washable Markers, 10-pack	Staples	$3.00