Networks of Trade and the Business Cycle

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Abstract

I explore the impact of the network structure of trade on business cycles using a simple model of interdependent production. In particular, how much amplification and co-movements does the network structure generate? I show that the network amplifies the volatility of production, generates a common business cycle and increases the volatility of prices. I also explore what is the best way to reduce the impact of network shocks through production stabilization of a given region. I find that the optimal tax base should include all regions directly or indirectly trading with such region.

Using data on the flow of commodities between american states, I test the predictions of the model and find results in line with the theory. I also use the model to estimate the probability of contagion between states using California and Texas as the "epicenter" of a downward shock. The network effect increases the probability of a downturn in direct trading states by 6%. Using world data, I argue that an Eurotax would be preferable to a local tax to insure against Greek downturns.

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1 Introduction

A network of trade can offer a convenient explanation to some unanswered question relating to the business cycles. Using a model that would otherwise be described as "perfect competition", I describe below that a fixed network interdependence amplifies the business cycle, generates positive comovements between countries and increases the volatility of the terms of trade.

By a network of trade, I mean the specification of required trade routes needed by region i in order to achieve its own production. For instance, United States imports oil from Canada, or any country that requires Microsoft Windows must import from the United States. These specifications of trade routes can be the outcome of complete product differentiation, as for the Microsoft Windows example, or the result of an institutional outcome. For instance, the United States tries to reduce its dependency on oil from overseas and therefore prefers imports from Canada. These trade specifications do not preclude from choosing the levels of imports, they only specify the allowed trade routes. Stated otherwise, they specify a network topology.

From a theoretical standpoint, the argument developed in this paper can also be seen as an exploratory one: in a model with perfect competition, how far can a fixed trade topology explain some business cycles facts? This paper does not adopt a particular point of view, but takes the network structure as a given, and embeds it in the form of technological requirements for production.

The direct corollary of a fixed trade topology is that if region j faces some business shock, the importing region i is necessarily submitted to it. It therefore follows that the network structure of imports influences the level of production in each region. In particular, it affects the volatility of production in other regions.

To understand how it affects volatility, consider a simple network of trade as given in Figure 1. There are only three regions of production and the input of each region is the output of a single other region as to form a simple trade cycle.

[Figure 1 about here[1].]

For instance, the production function of region one is given by:

$$y_{1t} = e^{\epsilon_{1t}} (y_{2t})^{1-\alpha}, \tag{1}$$

[1] Please refer to the Appendices for the figures and the tables.

and other production regions are defined likewise. For each region, there is a single demand and thus, for the purpose of this simple example, the price system and profit maximization decisions are moot. Given a particular realization of shocks $\epsilon_t = [\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}]'$, the solution to the system leads to:

$$y_{1t} = \exp\left(\epsilon_{1t} + (1-\alpha)\epsilon_{2t} + (1-\alpha)^2\epsilon_{3t} + (1-\alpha)^3\epsilon_{1t} + \dots\right).$$

If the variances on shocks are identical in each region, the variance of the log of each region is given by:

$$\mathrm{var}(\log(y_{1t})) = \frac{1 + (1-\alpha)^2 + (1-\alpha)^4}{(1-(1-\alpha)^3)^2}\sigma^2 > \sigma^2.$$

Under the same simplifying assumption on the variance, the covariance between region one and region three is given by:

$$\mathrm{cov}(\log(y_{1t}), \log(y_{3t})) = (1-\alpha)\mathrm{var}(\log(y_{1t})).$$

The fact that the covariance is positive, regardless for the flow of production, is a general result of the underlying model: it is solely a function of the "distance" of trade (measured in terms of the number of production regions in between) and is the driver for positive co-movements between any regions that are directly or indirectly connected.

The source of amplification is the consequence of two features of the model. First, the network topology. If a given production region needs the output of another region, there is necessarily some shock transmission through the chain of supply. Second, cycles of production. These cycles produce a feedback mechanism and thus, amplification.

The impact of the network can be understood through a decomposition in two components. First, the distributional effect of the network: how shocks of a given region are distributed over regions of production. Second, the feedback mechanism of cycles (as in the example above). The net effect depends on the interaction between these two effects. When regions with larger shocks lie on a cycle and that such cycle connects many regions of production, the amplification and distribution effect play in the same direction for other regions. When distributional effects mixes various shocks, or when the regions of a given cycle have a small variance, the effects partially offset each other. These two effects can be separated using the standard eigenvector/eigenvalue decomposition of the adjacency matrix of shares of imports between regions.

Given this particular mechanism for the transmission of shocks, a natural question is how to implement a stabilisation policy. I thus explore a stabilisation mechanism through optimal taxation. I find that that the optimal set of regions that should be insured against a volatile region should include no more than all regions importing directly or indirectly from it. If the network is fully connected, this means that taxation should include all production regions. If some regions are omitted, there is then a positive externality and under-provision of insurance. The optimal tax in each region depends on the magnitude of direct and indirect trade with the insured region. Hence, the less a region directly or indirectly imports from a given region, the less it should be taxed. This measure of direct or indirect trade can be used as a guide for the provision of insurance.

With the full model, I proceed to a a exercise using two datasets. In the first dataset, I use american states as regions of production. I first look at the data to find empirical results in line with the predictions of the model. Then, given the model assumptions, I proceed to an estimation of the statistical process in each state and ask the probability of a recession given that California and Texas, the most important state trade-wise, face a negative shock. A common shock of one standard deviation increases the probability of a downturn of 6% in adjacent trading states.

I also study countries as regions of production and ask how the Eurozone should insure itself against variations of production in Greece. I argue that an insurance scheme based on all countries in the Eurozone would be better off than local insurance. Although it is a practical Pareto improvement, it remains sub-optimal as it neglects the marginal gains of the insurance scheme outside Europe.

The rest of this paper is organized as follows. In section 1.1, I proceed to a brief literature review of business cycles and networks. In section 2, I present the model and some theoretical results. In section 3, I briefly present the datasets, some empirical predictions and the results of the estimation. A brief conclusion follows.

1.1 Literature Review

Empirical evidence as well as intuitive heuristics suggests that countries trading more with each other should have similar trends in terms of business cycles (see Baxter & Kouparitsas [5], Calderon & Al. [7], Clark & Wincoop [8], Frankel & Rose [11], Inklaar & Al. [12], [14], Otto & Al. [16] for a review). Standard models, when applied to trade, fail to reproduce this fact. They also fail to reproduce the volatility of the real exchange rate and predict a consumption volatility that is too high[2]. (See Backus, Kehoe &

[2] Various modifications have been introduced to improve upon these results. Some added transportation costs (Kose & Yi [14]) or vertical integration (Burstein, Kurz &

Kydland [13] or Baxter [4] for a general review).

Long & Plosser [15] introduced a model with multiple sectors of production and find amplification and persistance in line with some empirical results. It was recently used by Acemoglu, Carvalho, Ozdaglar & Tahbaz-Selehi [1] to discuss the impact of the network position of firms on the business cycle within a country[3]. In the international trade literature, Ambler, Cardia & Zimmerman [2] uses also a model with differentiated goods as intermediate producers and find results closer to the empirical facts. Based on their results, they argue however that Long & Plosser's replication of empirical results is based on the absence of capital accumulation in their model.

1.1.1 How This Article Contributes to the Literature

This paper contributes to the literature in two ways. First, it shows how a network topology can amplify the business cycle. Specifically, it shows that cycles of trade creates a feedback mechanism that increases the overall volatility.

Second, it introduces the study of a stabilisation mechanism against foreign shocks through the use of taxation. Unsurprisingly, it shows that pairwise insurance introduces externalities given the interconnectedness of the regions. It is thus sub-optimal. Pairwise insurance policy has a positive welfare effect for other regions because of the interconnectedness and thus, the externality is positive. I show that the best tax-base to insure against shock transmission of a given region are the other regions that are all connected directly or indirectly. Based on that, I argue that an European scheme is preferable to local taxation to insure against greek fluctuations.

2 Model

2.1 Notation

I use G to denote a **graph or a network** defined by a set of vertexes V and a single set of edges $E \subseteq V \times V$. Each vertex $i \in V$ is referred to as a **production region**, or simply a region. A directed edge $ij \in E$ denotes a flow of capital going from $j \in V$ to $i \in V$.

Tesar [6]), but such additions still fall short of explaining the anomalies.

[3] Most of their discussion about the "granular hypothesis" hinges around the redistributive effects of a network topology.

I denote production by the variable y. Thus, y_{it} represents the **total production** in region i and y_{ijt} denotes the capital **imports** by region i from region j at time t.

I append variables with a star to refer to the quantity in the steady state. Variables with a tilde on them refer to the log of the variable (or if it is a vector, the component-wise logarithm of such vector). The symbol Δ refers to the difference operator to the steady state and so, $\Delta\tilde{y}_{it}$ refers to $\log(y_{it}) - \log(y^*_{it})$. When the context is clear, I will refer to these variables as their original counterparts (production, prices, etc.) to lighten the text.

It is useful to denote the **neighbourhood demand** for capital of region i by $\eta_i \equiv \{j : ij \in E\}$, that is the regions selling a part of their production to region i. Close in spirit, the symbols s_{ij} are reserved to denote the **shares of input** for each region with the convention that $s_{ij} = 0$ if j is not an input for i ($j \notin \eta_i$). Equivalently, $S \equiv [s_{ij}]$ refers to the $|V| \times |V|$ matrix of shares.

Labor demand in region i is denoted by h_i while α, w_i are reserved for the **share of labor** and the **wage**. The utility of consumption is denoted $u(c_{it})$ with standard assumptions on u.

I use $'$ to denote the transpose of a vector or matrix. The matrix S' will often be decomposed in its eigenvector/eigenvalue form, which I will denote by[4] $V\lambda V^{-1}$.

Business shocks are denoted respectively $\epsilon_{it}, \boldsymbol{\epsilon}_t$ for a region specific or the whole vector of realizations. I use $L_i(\boldsymbol{\epsilon}_t)$ to denote a generic polynomial of lags on shocks. The selling price in region i and the vector of selling prices are respectively noted p_i and $\boldsymbol{p}$.

2.2 Model Setup

I consider a network $G(V, E)$ of regions of production where each region produces a single output according to a Cobb-Douglas form:

$$y_{it} = e^{L_{it}(\boldsymbol{\epsilon}_t)} h_{it}^{\alpha} \left(\prod_{j \in \eta_i} y_{ijt}^{s_{ij}} \right)^{1-\alpha}. \qquad \text{(Production Depends on Neighbours)}$$

This technological specification embeds the network topology, namely that the output of other regions in η_i are required inputs to produce. I also

[4]In the main body of the text, I avoid the notation for when V and λ are complex valued. Although both empirical networks studied below have a complex decomposition, I avoid most references to complex valued matrices to lighten the text. A complex valued decomposition has an economic interpretation related to stocks of production and natural fluctuations.

assume that the sum of shares s_{ij} sums to one. The production requires labor input h_{it}, which is assumed to be local (no migration). $L_i(\boldsymbol{\epsilon}_t)$ follows a stochastic process and $\epsilon_{it} \sim N(0, \sigma_i^2)$.

There is a single representative consumer in each of the regions of production. Such consumer does provide a fixed unit of labor in the region and receives a wage w_{it} for his supplied work. She consumes goods solely from the current region[5] and values consumption in the current period through the utility function u. For most of the paper, consumers do not save and have an "hand to mouth behaviour". This modelling choice of passive consumers is done solely to obtain a closed form solution. An extension of the model with savings and thus, capital accumulation, is discussed in Appendix A.3 to show that it does not affect the results about volatility.

A **competitive equilibrium** is a set of prices $\mathbf{p}(\boldsymbol{\epsilon}_t, G)$ wages $\boldsymbol{w}(\boldsymbol{\epsilon}_t, G)$, imports $y_{ijt}(\boldsymbol{\epsilon}_t, G)$ and consumption quantities $c_{it}(\boldsymbol{\epsilon}_t, G)$ such that firms maximize profits and supply equals demand:

$$\begin{aligned} y_{it}(\boldsymbol{\epsilon}_t, G) &= \sum_{j \in V} y_{jit}(\boldsymbol{\epsilon}_t, G) + c_{it}(\boldsymbol{\epsilon}_t, G) \ \forall i, \quad \text{(Market Clearing Conditions)} \\ 1 &= h_{it}(\boldsymbol{\epsilon}_t, G) \ \forall i. \end{aligned}$$

2.3 Solving The Model

Solving the model is straightforward and is done in Appendix A.1. The key trick lies in the market clearing condition, which can be written as

$$p_{it} y_{it} = (1-\alpha) \sum_{j \in V} s_{ji} p_{jt} y_{jt} + \alpha p_{it} y_{it}$$

Such system has the form $M_t = S' M_t$ where $M_t = p_t y_t$. Thus, M_t must be a multiple of the eigenvector associated with the leading eigenvalue of S' (which is a vector of ones).

An exact description of the economy can be summarized by the following

[5] Adding a second layer of edges on the network, each of them representing consumption imports, would not change much of the results.

five equations. The first four are in vector form.

$$\tilde{p}_t(\boldsymbol{\epsilon}_t, G) = -N(\tilde{a}^* + L(\boldsymbol{\epsilon}_t)), \tag{2}$$

$$\tilde{y}_t(\boldsymbol{\epsilon}_t, G) = N(\tilde{a}^* + L(\boldsymbol{\epsilon}_t)), \tag{3}$$

$$\tilde{c}(\boldsymbol{\epsilon}_t, G) = \alpha \tilde{y}_t(\boldsymbol{\epsilon}_t, G), \tag{4}$$

$$N \equiv (I - (1-\alpha)S')^{-1}, \tag{5}$$

$$[\tilde{a}^*]_i \equiv (1-\alpha) \sum_{j \in \eta_{iy}} \log((1-\alpha)s_{ij}). \tag{6}$$

The first equation describes the prices in terms of shocks, some steady state constant vector $\tilde{a}^*$ and the matrix $(I - (1-\alpha)S)^{-1}$. This matrix measures all the direct and indirect effects of a business shock on prices and can be understood as the cumulative effect of the network on production. To illustrate, consider a single shock in region i and for the sake of discussion, assume it is positive. The price of region i must account for the increase in production and so must account for the direct effect, accounted by the matrix $I = S^0$. This increase of production allows for an increase in production in adjacent regions as well, as they can import more and thus produce more. The log of these accrued sales is proportional to $(1-\alpha)s_{ji}$ for region j and so the vector of prices must account for $(1-\alpha)S$ for all importing regions, that is, the increase of production in the neighbourhood that is one step away from the production region. In turn, each of these regions in the neighbourhood will also profit from an increased production in the "neighbourhood of the neighbourhood", and so the price must also account for this effect through the term $(1-\alpha)^2 S^2$, that is regions two steps away from the production region. Repeating this argument infinitely often, the prices must account for the whole sum of accrued sales $I + (1-\alpha)S + (1-\alpha)^2 S^2 + \cdots = \sum_{t=0}^{\infty} (1-\alpha)^t S^t$. Recall that:

$$(I - (1-\alpha)S)^{-1} = \sum_{t=0}^{\infty} (1-\alpha)^t S^t, \tag{7}$$

which sums up the whole network effect.

Although the network is of finite dimension, summing up to infinity is important because prices must account for the feedback effect of cycles. As soon as one region is linked to itself through a trade cycle, prices are submitted the feedback effect of the cycle. The increase in production from other regions in the cycle will also boost demand and thus increase the production in the original region, as it allows for more input to be used.

Repeating this argument infinitely, each time weighted by the respective shares $(1-\alpha)s_{ij}$ over the cycle, yields the proper accounting for prices[6].

2.4 An Analysis of Amplification and the Covariance Between Regions

Equation 3 reveals that a production region imports business shocks from other regions, through the matrix N. And as such, it imports variance from other regions. This generates amplification and covariance between observed patterns of production.

The steady state can easily be found using the past equation. The difference equations can be summarized by:

$$\Delta\tilde{y}_t = NL(\boldsymbol{\epsilon}), \tag{8}$$

$$\Delta\tilde{p}_t = -NL(\boldsymbol{\epsilon}), \tag{9}$$

$$\Delta\tilde{c}_{ijt} = -\Delta\tilde{p}_{jt}. \tag{10}$$

Notice that N has the same eigenvector decomposition as S and its eigenvalue are the discounted sum of the eigenvalues of S:

$$N = \sum_{t=}^{\infty}(1-\alpha)^t S^t = V \underbrace{\sum_{t=0}^{\infty}(1-\alpha)^t \lambda^t}_{\equiv\Lambda} V^{-1}. \tag{11}$$

Now, consider a generic expression for the polynomial of lags in regions i:

$$L_i(\boldsymbol{\epsilon}_t) = \sum_{\tau=0}^{T} \alpha_{i\tau}\epsilon_{it-\tau}. \tag{12}$$

The covariance between regions of production can then be expressed in the following fashion:

$$\begin{aligned}\mathbb{E}\left[\Delta\tilde{y}_t\Delta\tilde{y}_t'\right] &= N\mathbb{E}\left[L(\boldsymbol{\epsilon}_t)L(\boldsymbol{\epsilon}_t)'\right]N' \\ &= N\begin{bmatrix} \sigma_1^2\sum_{\tau=0}^{T}\alpha_{it-\tau}^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2\sum_{\tau=0}^{T}\alpha_{it-\tau}^2 & \dots & 0 \\ \vdots & \vdots\ddots & \vdots & \\ 0 & 0 & \dots & \sigma_{|V|}^2\sum_{\tau=0}^{T}\alpha_{it-\tau}^2 \end{bmatrix}N'.\end{aligned} \tag{13}$$

[6] Recall that some eigenvalues of the matrix S are the geometric average of the shares of cycles on the edges. So in the example, one eigenvalue would be $((1-\alpha)s_{12}(1-\alpha)s_{23}(1-\alpha)s_{31})^{1/3} = (1-\alpha)\lambda_1$.

To understand how the topology amplifies, network, it is perhaps best to begin with two simplifying assumptions to get a neat characterization of the variance covariance matrix. So for the purpose of the explanation, assume that all lag of polynomials are identical to $\bar{L}$ and further assume that the network topology is symmetric. When the network is symmetric, is is a known result that $V^{-1} = V'$, which allows for the following simplification of the variance-covariance matrix:

$$\begin{aligned}\mathbb{E}\left[\Delta\tilde{y}_t \Delta\tilde{y}_t'\right] &= \sigma^2(\bar{L}) V \Lambda V' V \Lambda V', \\ &= \bar{L} V \Lambda^2 V'.\end{aligned} \tag{14}$$

Recall that the matrix S has all of its columns summing to one since each column represents the repartition of shares of production. It follows at once that the leading eigenvalue of S is equal to one. Hence, N has at least one eigenvalue greater than one since it accounts for the discounted sum, weighted by $(1-\alpha)$, of all eigenvalues:

$$\sum_{t=0}^{\infty}(1-\alpha)^t \lambda^t = \begin{bmatrix} \frac{1}{1-(1-\alpha)\lambda_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{1-(1-\alpha)\lambda_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{1-(1-\alpha)\lambda_V} \end{bmatrix}. \tag{15}$$

Following the simplifying assumptions for our discussion, this means that the variance on each region is given by:

$$\mathbb{E}\left[\Delta\tilde{y}_t \Delta\tilde{y}_t'\right] = V \begin{bmatrix} \frac{\sigma^2(\bar{L})}{(1-(1-\alpha)\lambda_1)^2} & 0 & \cdots & 0 \\ 0 & \frac{\sigma^2(\bar{L})}{(1-(1-\alpha)\lambda_2)^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\sigma^2(\bar{L})}{(1-(1-\alpha)\lambda_V)^2} \end{bmatrix} V'. \tag{16}$$

It is useful to think of the impact of N as the combination of the **distributional effect** of V and the **amplification effect** of Λ. This means that there is some amplification on each observed pattern of production if the amplification is large on cycles and such amplification is concentrated in regions through a weak distributional effect ($V_i \Lambda^2 V_i' > 1$).

Distributive effects are related to the concentration of connections to a single given cycle. As discussed in [1], a star topology, where all regions are connected only to one central region, gives a preponderant role to the shocks of that central region, maximizing the distributional effect. Conversely, a "balanced topology", where everyone is connected to everyone, minimizes the distributive effect by mixing shocks. As the example of figure 1 shown, amplification depends on the magnitude of the feedback effect on the cycles of the network.

Now, in the general case, the network might be symmetric and neither polynomials need to be identical, so the past decomposition remains a particular case. However, it is a known result of spectral decomposition that the determinant of a matrix is equal to the product of its eigenvalues. It is also known that for two square matrices the product of the determinant is the determinant of the product ($\det(AB) = \det(A)\det(B)$). It thus follows at once that the product of the eigenvalues of the variance-covariance matrix is nothing but the product of the eigenvalues of N (squared) and the diagonal matrix of variance of each polynomial. In other words, the total amplification on the variance covariance matrix is of the same order of magnitude as in the simplified case, but the distributional effects are different.

The covariance between nodes has a simple expression:

$$\operatorname{cov}(\Delta\tilde{y}_i, \Delta\tilde{y}_j) = \sum_{k\in V} [N]_{ik} \left(\sigma^k \sum_{\tau=0}^{T} \alpha_{k\tau}^2 \right) [N]_{jk}. \tag{17}$$

Such covariance is equal to zero only if there does not exist any path between regions of production ($N_{ij} = 0$). When this is so, both regions are in two disjoint graphs and none of the shocks can affect the other. Notice also that only a one way path is necessary to produce covariance between regions. If region i exports to region j and does not import anything, it will still import variance from the production of region j. This is so because for each flow of imports, there is an equivalent flow of income that goes to region i (through payments)[7].

Finally, covariance is always positive, regardless of the direction of flows. When the flow is in the direction of the imports, this is intuitive: variance is imported through the transfer of produced goods, which depends directly on business shocks. In the other direction, this is so because the flow of income depends negatively on business shocks and thus the two opposites cancel to yield a positive covariance. This is one crucial distinctive feature: although flows of imports are directed, co-movements between regions are not[8].

[7]Equation 3 also tells us that there are growth spillovers between regions of production. If shocks have a deterministic component in time to represent a pattern of fundamental growth, such growth will be shared proportionally to N_{ki}, the value of indirect imports from region k. Hence, growth in one region should also lead to growth in other regions.

[8]Notice also that the observed persistence in a given region can only be non-decreasing.

2.5 Stabilization Policy

If the network structure influences the volatility of production regions, a natural question is the optimal design of a stabilization policy so as to attenuate its effect. In this section, I develop this optimal insurance through the use of lump sum taxes to finance an insurance. I argue that the best tax base to insure against shocks of a given region needs to encompass all other regions importing directly or indirectly from this region. If there exists a directed path between any two regions (meaning the network is fully connected), this means that the set of all regions of production is the best tax base. The argument is simple: insuring against a fundamental risk has positive spillovers effect in all regions importing directly or indirectly because of the induced reduction in variance. If a government cannot insure all these regions, it cannot capture the correct benefits of the insurance. There is therefore, there is an under provision of insurance (or over taxation).

The easiest way to make such argument is to assume the existence of a benevolent government whose sole role is to insure a subset $I \subseteq V$ of consumers in production regions. This government offers a multiplicative production stabilizer $\exp\left((1-\phi)L_k(\boldsymbol{\epsilon}_t)\right)$ on region k. This stabilizer depends on $\phi \in [0,1]$, that is the degree to which a given shock is traded for certainty. The greater is such number, the more production in region k is stabilized. The stabilizer is funded through a lump-sum tax τ_{ki} in each insured region i.

Given this insurance scheme, the weighted expected welfare of consumers in I is given by:

$$\sum_{i\in I}\omega_i\mathbb{E}\left[u\left(\alpha\exp\left\{\sum_{i\in V}N_{ij}\left(\tilde{c}_j+L_j(\boldsymbol{\epsilon}_t)\right)-\phi N_{ik}L_k(\boldsymbol{\epsilon}_t)\right\}-\tau_{ik}\right)\right], \qquad (18)$$

where N_{ij} is the ij-th component of the matrix N and ω_i are the weights given to each region. The benevolent government seeks to maximize this welfare given an expected budget balance:

$$\sum_i \mathbb{E}\left[p_i\tau_{ik}\right]+\mathbb{E}\left[p_k y_{kt}-p_k y_{kt}\exp\left((1-\phi)L_k(\boldsymbol{\epsilon}_t)\right)\right]=0. \qquad (19)$$

Consider again the expression for the deviations of the (log) of GDP:

$$\Delta\tilde{y}_t = NL(\boldsymbol{\epsilon}_t).$$

The observed production at each period thus depends on a linear combination of the lag polynomials of each region. If some regions have polynomials longer than others, one will observe an increase in the persistance in regions with shocks of smaller length.S

Recall that a consequence of the equilibrium is $p_k y_k = 1$ for any region k, meaning that the budget constraint reduces to:

$$\sum_i \mathbb{E}\left[p_i\right] \tau_{ik} + 1 - \mathbb{E}\left[\exp\left((1-\phi)L_k(\boldsymbol{\epsilon}_t)\right)\right] = 0. \tag{20}$$

Denote L the lagrange multiplier associated with such problem. Aside from budget balance, the optimal tax scheme solves:

$$\omega_i \mathbb{E}\left[u'(\alpha y_{it} - \tau_{ki})\right] = L\mathbb{E}\left[p_i\right] \ \forall i, \tag{21}$$

$$\mathbb{E}\left[p_k\right] \sum_{i \in I} \omega_i \mathbb{E}\left[u'(\alpha y_{it} - \tau_{ik}) N_{ik} L_k(\boldsymbol{\epsilon}_t) y_{it}\right] = \omega_k \mathbb{E}\left[u'(\alpha y_{kt} - \tau_{kk})\right] \mathbb{E}\left[\exp\left((1-\phi)L_k(\boldsymbol{\epsilon}_t)\right) L_k(\boldsymbol{\epsilon}_t)\right]. \tag{22}$$

The first equation is a standard trade-off between efficiency and equity. If all regions had the same price, all expected marginal rates of utility, and thus all expected levels of consumption, should be equalized. Because expected utility depends negatively on variance of shocks, this means that more volatile regions are less taxed than less volatile ones. There is redistribution based on volatility. Now, expected prices are not equal, so there is an efficiency trade-off that is accounted by the value of production loss in region i, which is what the first equation is accounting for.

The second equation shows where the network externality comes in play. First, notice that if a region with $N_{ik} = 0$ is included in I, its contribution to the marginal benefit on the left-hand side of the equation is equal to zero. Therefore, adding such region to the tax base provides no change on the insurance in region k (and τ_{ik} would be used only for the purpose of redistribution). Recall that $N_{ik} = \sum_{j=0}^{\infty}(1-\alpha)^j [S'^j]_{ik}$ and is thus equal to zero if and only if there is no direct or indirect trade between i and k (e.g.: if the network is composed of at least two disjoint subgraphs, i being in one and k being in the other).

However, if a region i is connected to k, but not included in I, the left-hand side of the equation is then smaller, as all components are positive. This means that u' must be increased through an increase in taxation. Further, the right-hand side must also be decreased, so there is also a decrease in insurance provision. Hence, when a region is not included, taxation is increased and a smaller degree of stabilization is provided to the region. In particular, an insurance scheme provided by a local government is not socially efficient.

An indirect measure of the inefficiency can be measured by the sum of components N_{ik} outside I. If such sum is small, the social loss is unimportant while the converse is true if the remainder is large.

3 Empirical Estimation & Applications

I this section, I apply the model to two different datasets. The first one uses american states as regions of production, where I use the flow of commodities as a proxy for imports between states. I test various predictions of the model. First, I run a standard gravitational equation of trade between states, which can be derived from the model by including distance between states as part of the price. Then, I check if two states trading with each other exhibit co-movements in the GDP, as the theory predict they should. Second, I also check if there are co-movements for states that are indirectly trading with each other. The model predicts that there should be one, although the co-movements should be smaller. I then perform a structural estimate of the model to compute probabilities of recession in various states given negative shocks to California and Texas, the most two central states trade-wise.

The second datasets uses world countries as regions of production and bilateral trade data to model flow of imports between countries. As there has already been extensive empirical work on trade between countries, I focus on Europe and discuss what is the best european tax base to stabilize the effects of production shocks in Greece.

3.1 United States

3.1.1 Dataset

I use the Commodity Flow Survey to model the flow of different goods between states (Department of Transportation [9]). A particular observation of this dataset is accessed through a triplet [origin, destination, commodity code]. The commodity code key allows to specify a particular type of commodities in the total value of exports. As I focus on inputs in the production function, I use the commodity codes that broadly encompasses the notion of capital. Some examples are: Animal feed and products of animal origin (code 4), Monumental or building stone (code 10) or Coal (code 15). The full list is in Table 2 in Appendix.

When these three keys are specified, the dataset yields the value of the flow, in US dollars, for the given commodity in 2007. The dataset thus allows to build a network of supply and demand between states.

There are two main drawbacks with this dataset. First, for each triplet, there is only one observation, made in 2007[9]. Since shares of imports are based on these observation, there is only one observation to estimate the

[9] A new panel is about to be added in 2014.

shares s_{ij}. Second, commodities exclude products that are not physically moved from one state to another, like financial flows. So commodities are used as a proxy for all flows of imports.

I combine this dataset with the time-series of all 51 states' real Gross Domestic Product (Federal Reserve of St-Louis [10]). Each state has 16 annual observations, from 1997 to 2012. For each state, I take the log of the GDP and detrend it by using the Hodrick-Prescott filter with a bandpass of 6.25 and take the residual as $\Delta\tilde{y}_{it}$. As a value of α, I take the average of the ratio of wages payments to GDP in the US from 1947 to 2012, which is equal to 0.47. All parameters sources and calculations are summarized in Table 1.

[Table 1 about here]

3.1.2 Descriptive Figures

I first present some descriptive Figures of the network of trade between states. In Figure 2, I present the centrality graph of trade. I also present the heatmap of the matrices S and NN' in Figures 3 and 4. The variance covariance matrix NN' can be thought of the pure effect of the network on the variance, as if regions all had a variance of 1[10]

In Figure 2, the size of nodes depends of the size of production of the state it represents and the edges represent a flow of imports between two states. One can then see that California, Texas, Ohio and Pennsylvania are the biggest producers. The position of states measures the centrality in the import/export structure. The more a state is in the "center" of the graph, the more it has a central role in the whole system. California, Texas and Michigan are central trade-wise. The states of New York and New Jersey are also big producers, but they are less central as previous ones. Not surprisingly, Alaska, Hawaii and Washington DC are not central with respect to the flow of commodities. Iowa, Utah, Arizona, Kansas and Rhode Island figure amongst the smallest states in terms of production.

In this representation of the graph, the distance between states is also a measure that trade is more important between these states. For instance, one can see geographical clustering in the west part of the graph. Such region comprises most states of New England. Likewise, the southern region of the graph is mostly comprised of southern states (Florida, Alabama, etc.).

[Figure 2 about here]

[10] Alternatively, It can be thought as the effect on the variance measured in percentage.

Figure 3(a) shows an heathmap of the matrix of shares between states (the matrix S). The darker is an element ij, the more i imports from j. Since the diagonal is the darkest, the main element of the picture is that most states primarily trade with themselves. This fact however hides the trade patterns. If one restricts the scale of greys to be within a smaller range, as in Figure 3(b), trade patterns emerge. One can see which states export more than others, as the columns of theses states become darker than others. For instance, California, Illinois, Ohio and Texas are the largest exporters, followed by Indiana and Michigan. These are the most central nodes in Figure 2.

[Figure 3 about here]

I present in Figures 4(a), 4(b) and 5 various aspects of the matrix NN'. The first panel presents the heatmap of the variance covariance matrix while the last figure presents the main diagonal. The network effect of the United States seems to mainly have an impact on variance as the main diagonal of S remains the most important source of trade. Scaling the heatmap however reveals that the patterns of trade have a direct incidence on the covariance between states. The most central states, California and Texas, should exhibit the highest correlation with other states.

[Figure 4 about here]

[Figure 5 about here]

3.1.3 Gravitational Trade and Estimates of Co-Movements

In this section, I look for empirical evidence of the model built in the previous section. In the mathematical appendix, I show that imports, or the demand by region i for the product in region j is given by:

$$y_{ij} = (1-\alpha)s_{ij}\frac{p_i}{p_j}y_i \tag{23}$$

using the relationship for prices, this expression is equivalent to:

$$y_{ij} = (1-\alpha)s_{ij}y_iy_jp_i \tag{24}$$

If one is ready to let the prices be a function of distance (for instance, if $p_{ij} = p_jD_{ij}$), this equation becomes:

$$\tilde{y}_{ij} = (1-\alpha)s_{ij}\frac{y_iy_jp_i}{D_{ij}} \tag{25}$$

which is a standard gravitational model of trade. Using observations from 2007, I run the non-linear least-squares on :

$$y_{ij} = e^{\beta_0} y_{it}^{\beta_1} y_{jt}^{\beta_2} \mathrm{dist}_{ij}^{\beta_3} + e_{ij}, \tag{26}$$

The non-linear specification is used to avoid biaises in log form due Jensen's Inequality (see **??**). I present the results in Table 3.

[Table 3 about here]

Aside from the significance of the results, the order of magnitude of coefficients on GDP seems in line with the suggested value given by the model (1) but are however significantly different. Taken together, this estimation suggest there is some grounds for the current model of imports.

I look at co-movements of GDP between states. This is done using standard maximum likelihood. I first focus on states who have an important share of imports from a another state and look if deviations from the steady state in the producing state can explain the deviations in the importing state. An example of the regressions I ran for each pair of state is given in Table 4. In theory, the ability to explain deviations in state i from deviations in state j depends on the component N_{ij}. Recall that if there is amplification by the network, such component should be greater than the value of the share of imports $(1-\alpha)s_{ij}$. Further, if the network is fully connected, such component should be non-zero.

[Table 4 about here]

I report in Table 5 a summary of the regressions ran. The results found that co-movements are aligned with the theory. The estimated coefficients are significantly greater than their predicted share of imports, which suggest that there is amplification over the network of trade. Only Louisiana shows no relationship with its main source of importation. One possible cause is Hurricane Katrina, event that happened during the period of study.

[Table 5 about here]

As the network of trade of american states is fully connected, the theory predicts that states that are not directly connected should also be able to explain co-movements of these states (because N_{ik} is not equal to zero). At the bottom of the table, I report the coefficients of co-movement between Hawaii and Eastern states. All of the coefficients are smaller and positive, as they are only indirectly connected.

Two states that do not behave as predicted: Alaska and the District of Columbia. Both of them show significant negative co-movements with other states, including importing states. In the case of Alaska, I conjecture two effects. First, its main partner in trade for commodities is probably the province of British Columbia, Canada. Second, distance might be sufficient to introduce important lags in the propagation of the business cycle (as suggest regressions with lagged deviations of continental states, although unsignificant). The District of Columbia is the siege of the government. It might possible that its economic activities is linked to governmental spending, which is counter-cyclical.

3.1.4 Shock Propagation

In this section, I move on to a structural estimation of the model. That is, given the matrix N, I estimate $L_i(\boldsymbol{\epsilon}_t)$. The most important application to such equations is the ability to compute the propagation of shocks between regions. Given a contemporaneous shock in region j, what is the expected contemporaneous impact on region i? I illustrate the interest by looking at the impact of an adverse shock to both California and Texas, the two most central states in the United States. I then look at the probability that other states show a decrease in output.

Finding $L_i(\boldsymbol{\epsilon}_t)$ needs to be programmed and given the number of observations for each state, I restrict my attention to autoregressive models of order three or less. For each region estimated, I keep the model that has all of the coefficients significant and that has the highest number of them.

The estimation technique is equivalent to maximum likelihood on the series $\Delta\tilde{z}_{it} \equiv V^{-1}\Delta\tilde{y}_{it}$, where V^{-1} is the matrix of eigenvectors generated by the matrix S. By construction, each variable $\Delta\tilde{z}_{it}$ is independent of each other, with variance $\sigma_i^2/\left(1-(1-\alpha)\lambda_i\right)^2$. There is only one catch: V^{-1} lies in the complex space. In appendix A.2, I show however that coefficients of such estimation are real and their estimated value can be found using the untransformed dataset. The variance needs however to be corrected by the given transformation. I report in Tables 7 to 17 the resulting estimation for each state.

Recall that effective shocks on a region i are given by :

$$\Delta\tilde{y}_{it} = \sum_{i\in V} N_{ik}\epsilon_{kt}.$$

The probability of a decrease of output given that California has a negative shock:

$$\begin{aligned}
&\mathbb{P}\left(\Delta\tilde{y}_{it} < 0 \middle| \epsilon_{CAt}, \epsilon_{TXt}\right) \\
&= \mathbb{P}\left(\sum_{k \in V\setminus\{CA,TX\}} N_{ik}\epsilon_{kt} < -\left(N_{iCA}\epsilon_{CAt} + N_{iTX}\epsilon_{TXt}\right) \middle| \epsilon_{CAt}, \epsilon_{TXt} \right).
\end{aligned}$$

By assumption of independence on each error term. the statistic $\sum_{k\in V\setminus\{CA,TX\}} N_{ik}\epsilon_{kt}$ follows a normal distribution $N(0, \sum_{k\in V\setminus\{CA,TX\}} N_{ik}^2\sigma_k^2)$ and thus the probability of interest is given by:

$$\Phi\left(-\frac{\left(N_{iCA}\epsilon_{CAt} + N_{iTX}\epsilon_{TXt}\right)}{\sqrt{\sum_{k\in V\setminus\{CA,TX\}} N_{ik}^2\sigma_k^2}} \right),$$

where Φ is the cumulative normal distribution.

I report in Table 6 the states who are the most vulnerable to such shocks for given multiples of standard deviations on both Texas and California. The full table can be found in Appendix (Table 18).

[Table 6 about here]

The table shows that a joint downward shock of one standard deviation on both states increases the chances of a downturn in these selected states by roughly 6%. The most vulnerable states are of course Texas and California themselves with an increase from 50% to respectively 74% and 67%. Such probabilities are not equal to one because there is still a chance that states from which they import can sustain their decrease of production. Other adjacent states see a moderate increase in the probability of a downturn.

3.2 World Data

3.2.1 Dataset

I use the Correlates of War dataset of bilateral trade [3] to model the flow of trade between world countries. A particular observation is accessed through a triplet (origin, destination, year). Each observation represents the current value of imports in US dollars from "origin" to "destination" for a given year. Each observation has been modified to correct for inflation using the current price index. The shares s_{ij} have been estimated as the average of share of observations from 1950 to 2009.

The dataset has been combined with the UN aggregate accounts for each country to obtain the share of production that remains in the country (e.g. the production that is not exported), yielding the diagonal component of S. The parameters employed for this section can be found in Table 19.

[Table 19 about here.]

3.2.2 Descriptive Statistics

I present in Figures 6 and 7 respectively the centrality graph and the heatmap of S'. United States is the most central country surrounded by Bresil, Belgium, Philippines, South Africa and Canada. The only reason Canada is at the center is its indirect importance through its main trade partner, United States, as trade with almost nobody else. Most countries of the G8 are not far from the center, like Germany, Spain, Japan, United Kingdom and France. China, is also central.

Some countries are also dominant exporters (relative to their GDP), as the heatmap shows: Japan (col. 14), Netherlands (col. 23), France (col. 45), the United States (col. 56), China (col. 58), Russia (col. 72), Italy (col. 133), Germany (col. 148) and United Kingdom (col. 168).

In Figures 8 and 9, I present the impact of the network of trade on variance and covariance between countries. Unsurprisingly, most variations are correlated with the United States, China, Japan and Germany. The amplification effect of the network on variance is high, and the correlation between countries is much higher than between american states. In particular, there is a net correlation with the United States for most countries.

Singapore (col. 166) and the Luxembourg (col. 92) however exhibit a decrease in variance. Otherwise, the network generates an higher variance.

3.3 (Sub) Optimal Insurance For Greece In the Eurozone

In this section, I investigate the optimal tax scheme against variations in the Greek production. Recall that the optimal tax base should include all regions that are directly or indirectly connected to Greece. As the world is fully connected, the optimal tax base cannot be implemented. I explore how a tax in the Eurozone can improve the insurance scheme. I report in table 20 the cumulative sum of the indirect imports N_{ij} of adding a country in the tax base for each country in the Eurozone.

Recall that this cumulative sum measures the relative gain in efficiency of adding a given region to the tax base. The first country to be added is of course Greece itself, as it benefits the most from insuring itself. The other countries that benefits the most from the insurance are Cyprus, Germany, Italy and France. Adding all countries in the Eurozone yields a coverage of 69% of the optimal tax base. The marginal benefit of each country outside the Eurozone is relatively small, but as there is a large number of them, the inefficiency of not including them remains important.

[Table 20 about here]

4 Conclusion

In this paper, I have argued that trade rigidities can help explaining both co-movements between countries as well as the volatility of the terms of trade. I have used a model that embeds these trade rigidities as technological requirements (inputs) for any region to produce. In this context, a trade network acts both as a mechanism for transmission of the business cycle but also as a feedback mechanism, generating amplification.

Based on this model, I have argued that the best insurance scheme against fluctuations in a given region, taking Greece as an example, is better when the insurance scheme encompasses all countries directly or indirectly importing from that region. This is so because trade creates a network externality and countries not directly trading with other benefits from stability through the stability imported from their indirect partners. Using Europe as an exemple, I argue that an Eurotax is preferable to any pairwise insurance system.

I have also used american states as regions of production and explored how well the model can explain some empirical facts. The model can explain the relationship of standard gravity models. It further predicts that closer states, in terms of trade, should have an higher covariance between regions of production. This result holds regardless of the direction of the trade flows. I finally used the model to make a structural forecast of the impact of a downward shock to California and Texas on other states. I find that adjacent states face a moderate decrease (6% decrease) of their production for a downward shock of one standard deviation.

A Mathematics

A.1 Solving the Model

The profit maximization problem is given by:

$$\max_{y_{ijt},h_{it}} p_{it}A_{it}h_{it}^{\alpha}\left(\prod_{j\in\eta_i} y_{ijt}^{s_{ij}}\right)^{1-\alpha} \tag{Profit Maximization}$$

$$\text{s.t.} M_{it} = \sum_{j\in\eta_i} p_{jt}y_{ijt} + w_i h_{it}$$

$$M_{it} = p_{it}y_{it}$$

This leads to the following first-order conditions:

$$(1-\alpha)s_{ij}\frac{p_{it}}{p_{jt}}y_{it} = y_{ijt}\ \forall i,j \tag{y_{ijt}}$$

$$\alpha\frac{p_{it}}{w_{it}}y_{it} = h_{it}\ \forall i \tag{h_{it}}$$

Define the constant a_i^* by:

$$a_i^* \equiv \left(\prod_{j\in\eta_i}(1-\alpha)s_{ij}\right)^{(1-\alpha)s_{ij}},$$

and combine the first order conditions to obtain:

$$y_{it}(\boldsymbol{p},G) = A_i^{\frac{1}{\alpha}} {a_i^*}^{\frac{1}{\alpha}} p_i^{\frac{1-\alpha}{\alpha}} \prod_{j\in\eta_i} p_j^{\frac{1-\alpha}{\alpha}s_{ij}} \forall i \tag{Indirect Production}$$

$$y_{ijt}(\boldsymbol{p},G) = (1-\alpha)s_{ij}\frac{p_i}{p_j}y_i,\ \forall i,j \tag{Demand for Good j by Region i}$$

$$(1-\alpha)y_{it}(\boldsymbol{p},G) = \sum_{j\in V} s_{ji}y_{jit}(\boldsymbol{p},G)\ \forall i \tag{Market Clearing Condition}$$

If we multiply the market clearing condition, we obtain:

$$p_i y_{it} = \sum_j s_{ij}p_j y_{jt},$$

$$\Leftrightarrow \quad M = S'M.$$

This last expression means that income produced by each region must be a fixed point of the network topology. This implies that M must be a multiple of the eigenvector of S' associated with the eigenvalue 1. Since the sum of shares (rows in S') sums to one, such eigenvalue is guaranteed to exist and the eigenvector is given by $\imath = [1, 1 \ldots, 1]$ (recall that $S'\imath = \imath$ by construction since each row sums to one). Since any multiple of an eigenvector is also an eigenvector, the income in all regions is equal to some scalar function $f(\boldsymbol{p}, G)$ (which can be equal to one). In particular, this implies that:

$$p_i y_i = p_j y_j \ \forall i, j.$$

I use this additional equation and the degree of liberty of the price system to fix $p_1 x_1 = 1$. With the indirect production function, this yields a set of $|V|$ equalities:

$$A_i^{\frac{1}{\alpha}} a_i^{*\frac{1}{\alpha}} p_i^{\frac{1}{\alpha}} \prod_{k \in \eta_i} p_k^{\frac{1-\alpha}{\alpha} s_{ik}} = 1,$$

or, expressed in logs:

$$0 = \tilde{A}_{it} + \tilde{a}_i^* + \log(p_{it}) - (1-\alpha) \sum_{j \in \eta_i} s_{ij} \log(p_{jt}) \ \forall i$$

In matricial terms, this means:

$$\begin{aligned} & 0 = \tilde{A}_t + \tilde{a}^* + (I - (1-\alpha)S)\tilde{p}_t \ \forall i \\ \Rightarrow \quad & \tilde{p}_t = -(I - (1-\alpha)S)^{-1}(\tilde{A}_t + \tilde{a}^*) \end{aligned}$$

If we plug back this solution in the indirect production function (in logs), one then gets:

$$\tilde{y}_t = (I - (1-\alpha)S)^{-1}(\tilde{A}_t + \tilde{a}^*).$$

A.2 Maximum Likelihood on a Complex Dataset

In such model, the series $V^{-1}\Delta\tilde{y}_{it}$ is statistically independent of shocks in other regions. It can be thus used to estimate $L_i(\boldsymbol{\epsilon}_t)$, the underlying statistical process of each region. If the network is directed, nothing prevents V^{-1} to lie in $\mathbb{C}^{|V|^2}$, meaning that the network as a natural, cyclical component[11].

For instance, the following matrix of shares:

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

[11] The interpretation of the imaginary

represents the simple network in figure 1. Its eigenvector decomposition is given by:

$$V = \begin{bmatrix} \frac{\sqrt{0.75}}{3} + 0.5i & -\frac{\sqrt{0.75}}{3} - 0.5i & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\sqrt{0.75}}{3} - 0.5i & -\frac{\sqrt{0.75}}{3} + 0.5i & \frac{1}{\sqrt{3}} \end{bmatrix},$$

and therefore, its orthogonal times series has a complex representation. Notice further that the decomposition of S is now given by $S = V\Lambda\bar{V}$, where $\bar{V}$ is the complex conjugate of V. Consider the following generic transformation $\bar{V}y = \bar{V}X\beta + \bar{V}\epsilon$, where $\bar{V}\epsilon$ has independent realizations in the complex space with variance $\mathbb{E}\epsilon'\bar{V}V\epsilon = \sigma^2$. The log-likelihood of an observation is given by:

$$l(\epsilon_i) = \frac{1}{2}\log\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2}(\bar{V}y - \bar{V}X\beta)'\overline{\left(\bar{V}y - \bar{V}X\beta\right)}.$$

Expanding $\beta = \Re(\beta) + \Im(\beta)i$, one can then show that:

$$\Re(\hat{\beta}) = (X'X)^{-1}X'Y,$$
$$\Im(\hat{\beta}) = 0.$$

Hence, if the original series has no complex part, the estimated coefficients will be the same as running the coefficients under the original series (and estimated betas have a null imaginary part).

A.3 Variance on Savings and Capital Accumulation

A criticism of the model presented above is the lack of capital accumulation, or a depreciation rate of capital equal to 100%. Ambler, Cardia & Zimmerman argued that the main driver for amplification is not a trade topology but rather the lack of capital accumulation. In this section, I however show that the network effect still amplifies business cycles. I further show that the unconditional variances of production when there is accumulation is in fact equal to when there is full depreciation. This happens simply because capital accumulation depends on past shocks in other regions. Capital accumulation thus introduces lags, but does not change the main driver for amplification.

To understand the point, some modifications to the model are required. First, production at time t in region j depends on the accumulated capital of assets K_{ijt}. This capital is interpreted as fixed physical capital[12] (as a factory for instance) and pertains to the region i. I assume a constant rate of depreciation δ across all kinds of capital to simplify the exposition. Hence, for each type of capital in each region, the law for capital accumulation is given by:

$$K_{ijt} = (1-\delta)K_{ijt-1} + I_{ijt} \ \forall i, j \tag{27}$$

where I_{ijt} is the investment made by region i in capital produced by region j at time t.

Capital is held by the consumers in the region of production and paid with an interest rate r_{ijt}. Thus, transactions are no longer between pair of firms, but between a firm and a capital owner. I assume that consumers save a constant fraction ϕ_{ij} of their wage and thus, $I_{ijt} = \phi_{ij}\frac{p_{it}}{p_{jt}}y_{it}$. Notice that for the consumer to have anything to consume, it must be that $\sum_j \phi_{ij} < 1$ for all j, so the matrix $\Phi \equiv [\phi_{ij}]$ has a leading eigenvalue smaller than one.

With this in mind, I show below that the unconditional variance on production is approximated by the following system of equations:

$$\mathrm{var}\left(\Delta\tilde{y}_{it}\right) = \sigma_i^2 + \sum_{j\in\eta_i}(1-\alpha)^2 s_{ij}^2 \mathrm{var}\left(\Delta\tilde{y}_{jt}\right) \ \forall i, \tag{28}$$

which is nothing but a linear system whose solution amplifies the variance of business shocks. This means that the system still faces the same amplification on variance, regardless of the depreciation rate. This is so because capital depends on the history of shocks as well and thus, its level is still influenced by the shocks in the importing regions (although past ones). Since such results depends on the adjustment of capital, this also introduces persistance in the patterns.

The main steps for solving this variant of the model are similar to the main model. The market equilibrium condition is given by the following equation:

$$y_{it} = \sum_{j\in V}\phi_{ji}\left(\frac{w_{jt}L_{jt}}{p_{it}} + \sum_{k\in V}\frac{r_{jkt}K_{jkt}}{p_{it}}\right) + \left(1 - \sum_i \phi_{ij}\right)\frac{w_{it}L_{it}}{p_{it}}. \tag{29}$$

This leads, in a matrix algebra formulation, to the following system:

$$M_t = \left(I - \Phi_1\right)^{-1}\Phi' M_t,$$
$$M_{it} = p_{it}y_{it}$$

[12]The location of the capital is fixed. The ownership of the capital does not need to be local and could be allowed across a network of capital flows. This only affect production prices and is thus left aside.

and thus, M_t is a multiple of the eigenvector of $(I - \Phi_1)^{-1}\Phi'$ associated with the unit eigenvalue. Because production is also known at every period, this means there is still a perfect contemporaneous correlation between prices and production since $v_i = p_{it}y_{it}$.

Now, the fluctuations for the production around the steady state is given by:

$$\Delta\tilde{y}_{it} = \epsilon_{it} + \sum_{j\in\eta_i}(1-\alpha)s_{ij}\Delta\tilde{K}_{ijt}, \tag{30}$$

which implies that:

$$\mathrm{var}\left(\Delta\tilde{y}_{it}\right) = \sigma_i^2 + \sum_{j\in\eta_i}(1-\alpha)^2 s_{ij}^2 \mathrm{var}\left(\Delta\tilde{K}_{ijt}\right) \ \ \forall i \tag{31}$$

An approximation on $\Delta\tilde{K}_{ijt}$ is given by a Taylor development of the equation for capital accumulation around the steady state:

$$\begin{aligned} K_{ijt} &= (1-\delta)K_{ijt-1} + \phi_{ij}\frac{p_{it}y_{it}}{p_{jt}}, \\ &= (1-\delta)K_{ijt-1} + \phi_{ij}\frac{v_i}{v_j}y_{jt}, \end{aligned} \tag{32}$$

$$\begin{aligned} \Rightarrow \Delta\tilde{K}_{ijt} &\approx (1-\delta)\frac{\left(K_{ijt-1} - K_{ij}^*\right)}{K_{ij}^*} + \phi_{ij}\frac{v_i}{v_j}\frac{y_j^*}{K_{ij}^*}\frac{\left(y_{jt} - y_j^*\right)}{y_j^*}, \\ &\approx (1-\delta)\Delta\tilde{K}_{ijt-1} + \phi_{ij}\frac{v_i}{v_j}\frac{y_j^*}{K_{ij}^*}\Delta\tilde{y}_{jt}. \end{aligned} \tag{33}$$

Notice further that at the steady state, we have:

$$\delta K_{ij}^* = \phi_{ij}\frac{v_i}{v_j}y_j^*, \tag{34}$$

and this can be used to find:

$$\Delta\tilde{K}_{ijt} \approx (1-\delta)\Delta\tilde{K}_{ijt-1} + \delta\Delta\tilde{y}_{jt}, \tag{35}$$

stationarity around the steady state implies:

$$\delta^2\mathrm{var}\left(\Delta\tilde{K}_{ijt}\right) \approx \delta^2\mathrm{var}\left(\Delta\tilde{y}_{jt}\right). \tag{36}$$

Thus the variance on production can be approximated by the following around the steady state:

$$\mathrm{var}\left(\Delta\tilde{y}_{it}\right) = \sigma_i^2 + \sum_{j\in\eta_i}(1-\alpha)^2 s_{ij}^2 \mathrm{var}\left(\Delta\tilde{y}_{jt}\right) \ \ \forall i. \tag{37}$$

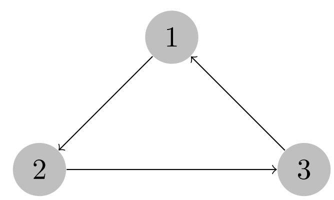


Figure 1: **A Simple Network of Shock Propagation**

B Figures & Tables

Table 1: Parameter and Sources for the United States Network

Symbol	Value	Calculation	Source
α	0.47	$\frac{1}{T}\sum_{t=0}^{T}\frac{w_t L_t}{Y_t}$	US GDP and Wages and salaries (FRED).
s_{ij}	N/A	$\frac{\sum_{k\in\text{codes}} p_{jk}y_{jk}}{\sum_{j\in\eta_i}\sum_{k\in\text{codes}} p_{jk}y_{jk}}$ for $j \in \eta_i$.	Commodity Flow Survey (Dept. of Transportation)

Commodity Code	Commodity	Capital ?
0	All Commodities	No
1	Live animals and live fish	No
2	Cereal grains	No
3	Other agricultural products	Yes
4	Animal feed and products of animal origin.	Yes
5	Meat, fish, seafood, and their preparations	No
6	Milled grain products, preparations and bakery products	No
7	Other prepared foodstuffs and fats and oils	Yes
8	Alcoholic beverages	No
9	Tobacco products	No
10	Monumental or building stone	Yes
11	Natural sands	Yes
12	Gravel and crushed stone	Yes
	...	

Commodity Code	Commodity	Capital ?
	...	
13	Nonmetallic minerals	Yes
14	Metallic ores and concentrates	Yes
15	Coal	Yes
16	Inexistant	No
17	Gasoline and aviation turbine fuel	Yes
18	Fuel oils	Yes
19	Coal and petroleum products	Yes
20	Basic chemicals	Yes
21	Pharmaceutical products	No
22	Fertilizers	Yes
23	Chemical products and preparations	Yes
24	Plastics and rubber	Yes
25	Logs and other wood in the rough	Yes
26	Wood products	No
27	Pulp, newsprint, paper, and paperboard	Yes
28	Paper or paperboard articles	Yes
29	Printed products	No
30	Textiles, leather, and articles of textiles or leather	No
31	Nonmetallic mineral products	Yes
32	Base metal in primary or semifinished forms [...]	Yes
33	Articles of base metal	No
34	Machinery	Yes
35	Electronic [...], electrical equipment and [...] office equipment	Yes
36	Motorized and other vehicles (including parts)	Yes
37	Transportation equipment	Yes
38	Precision instruments and apparatus	Yes
39	Furniture, mattresses, [...] lamps [...]and illuminated signs	No
40	Miscellaneous manufactured products	No
41	Waste and scrap	Yes
42	Inexistant	No
43	Mixed freight	No

Table 2: **Commodity Codes and Their Inclusion As Capital**

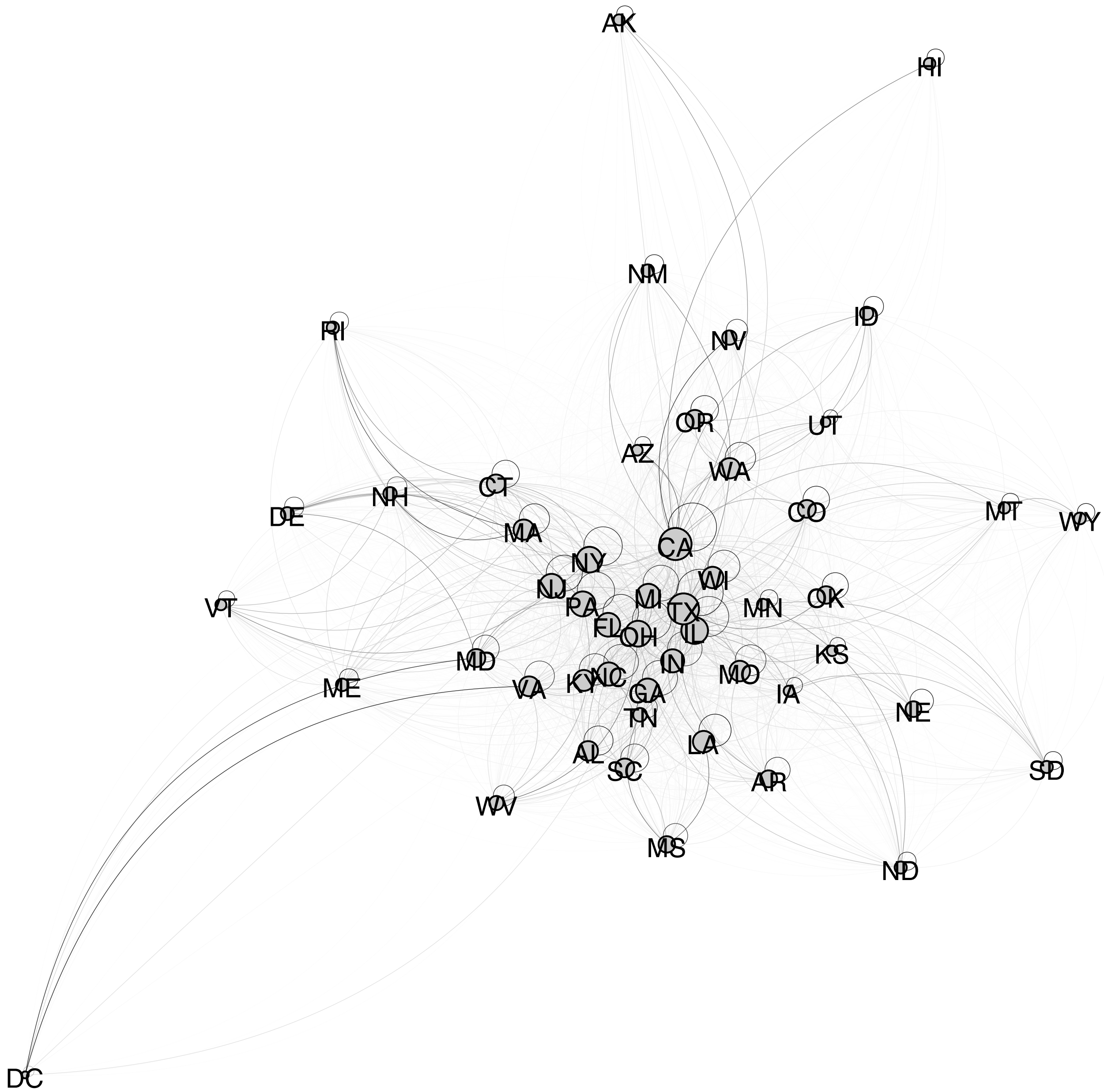
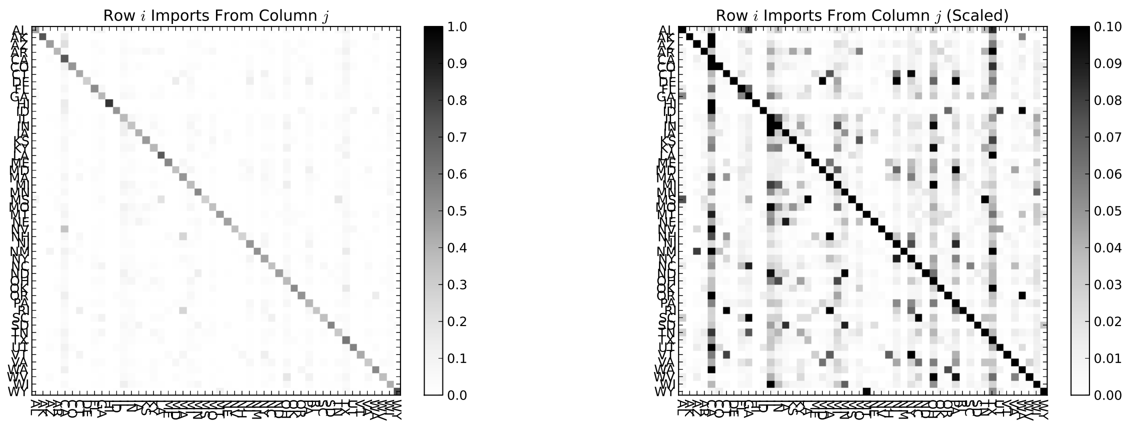


Figure 2: **Network Of Capital Flow Between US States.**

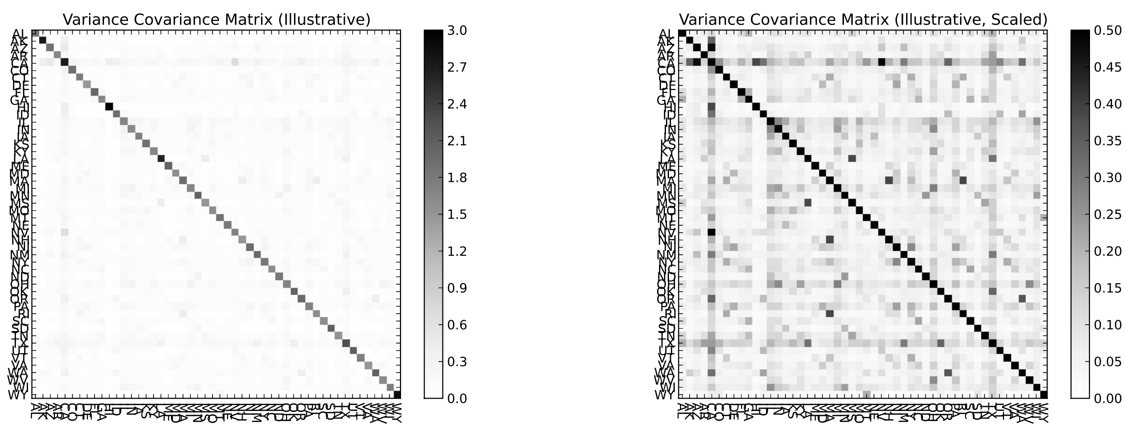
Source: Commodity Flow Survey (2010), Department of Transportation and calculation. Network displayed according to the Fruchterman-Reingold layout. The repulsive force is proportional to the fourth root of production and a bounding box of 700pts × 700pts is used. Each node has a size proportional to the fourth root of production. The higher the share of trade, the darker are the edges. Representation produced with Python and iGraph.



(a) **Original Data** (b) **Colors Scaled**

Figure 3: **Heatmap of** S'

Source: Commodity Flow Survey (2010), Department of Transportation. Each share of capital is computed as the ratio of imports to the total ratio of imports. Produced with Python, iGraph and PyPlot. In the second panel, the colormap is restricted between $[0, 0.1]$ to exhibit exporting states (dark columns).



(a) **Original Data** (b) **Colors Scaled**

Figure 4: **Heatmap of** $(I-(1-\alpha)S)^{-1}(I-(1-\alpha)S)^{-1'}$

Source: Commodity Flow Survey (2010), Department of Transportation and calculation. Produced with Python, iGraph and PyPlot. In the second panel, the colormap is restricted between $[-0.5, 0.5]$ to exhibit covariance.

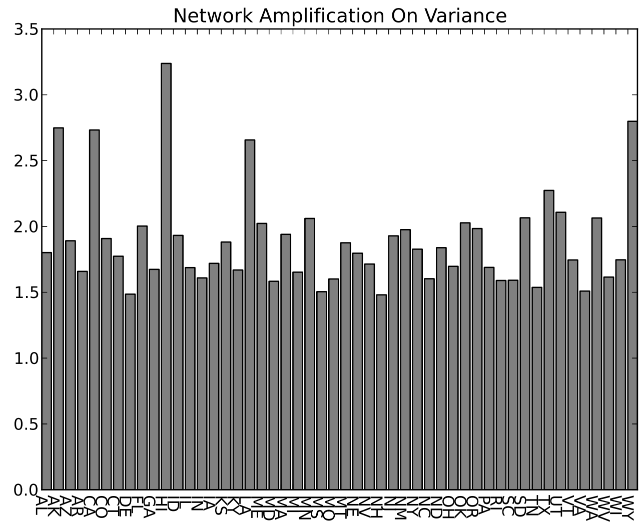


Figure 5: **Main Diagonal of** $(I-(1-\alpha)S)^{-1}(I-(1-\alpha)S)^{-1'}$ **(Variance)**

Source: Commodity Flow Survey (2010), Department of Transportation and calculation. Produced with Python, iGraph and PyPlot.

y_{ij}	Coefficient
log(Cons)	-24.66***
	(0.676)
y_i	1.340***
	(0.0504)
y_j	1.423***
	(0.0514)
Dist_{ij}	-0.345***
	(0.00958)
Observations	1,859
R-squared	0.620

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 3: Results of a Gravitational Equation Estimation

	(1)	(2)	(3)
$\Delta\tilde{y}_{\mathrm{WA}t}$			
$\Delta\tilde{y}_{\mathrm{CA}t}$	0.803***	0.852***	0.852***
	(0.279)	(0.262)	(0.258)
Constant	5.33e-05	0	
	(0.00135)	(0.00147)	
$\Delta\tilde{y}_{\mathrm{WA}t-1}$	0.116		
	(0.399)		
$\Delta\tilde{y}_{\mathrm{WA}t-2}$	-0.277		
		(0.499)	
$\hat{\sigma}$	0.00468***	0.00491***	0.00491***
	(0.00105)	(0.000960)	(0.000901)
Observations	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Source: Federal Reserve Data. Estimations produced with Stata.

Table 4: Typical Relationship of Co-Movements Between Two Trading Regions (Here, Washington and California)

i	j	Coeff.	Share of Imports ($(1-\alpha)s_{ij}$)	Amplification?
WA	CA	0.852***	0.1157	yes
UT	CA	0.637***	0.1024	yes
OR	CA	0.802***	0.1200	yes
NM	CA	0.091	0.1082	no
NV	CA	1.361***	0.3445	yes
MT	CA	0.375***	0.0900	yes
ID	CA	0.969***	0.1172	yes
AZ	TX	1.366***	0.0586	yes
CO	TX	0.680*	0.0841	yes
LA	TX	-0.483	0.1068	no
OK	TX	0.491*	0.1619	yes
IN	IL	1.413***	0.114	yes
LA	IL	-0.292	0.0144	no
MO	IL	0.626***	0.0983	yes
ND	IL	0.145	0.0946	yes
WI	IL	0.871***	0.1211	yes
IN	OH	1.144***	0.0958	yes
MI	OH	1.460***	0.1173	yes
WV	OH	0.345	0.1782	yes
VT	HI	0.502****	0.0	yes
VA	HI	0.384***	0.0	yes
NY	HI	0.475***	0.0	yes

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Source: Federal Reserve Data. Estimations produced with Stata.

Table 5: Coefficient Explaining Co-Movements Between State i and j

State	0	$-\sigma/2$	$-\sigma$	$-3/2\sigma$	-2σ
CA	0.5000	0.5449	0.5891	0.6323	0.6739
CO	0.5000	0.5136	0.5272	0.5407	0.5542
FL	0.5000	0.5153	0.5306	0.5458	0.5609
NE	0.5000	0.5135	0.5269	0.5404	0.5537
NV	0.5000	0.5142	0.5283	0.5424	0.5565
NH	0.5000	0.5136	0.5271	0.5406	0.5541
OK	0.5000	0.5130	0.5259	0.5388	0.5517
TX	0.5000	0.5647	0.6277	0.6875	0.7427
WA	0.5000	0.5140	0.5280	0.5419	0.5558
WV	0.5000	0.5151	0.5302	0.5453	0.5603
WI	0.5000	0.5222	0.5443	0.5662	0.5880

Table 6: Probability That State i Dives For A Given Dive in Texas and California (Selection)

EQUATION	VARIABLES	AK	AL	AR	AZ	CA
ARMA	L.ar				0.754***	0.643***
					(0.192)	(0.234)
	L2.ar				-0.640***	-0.603*
					(0.185)	(0.316)
ARMA	L3.ar			-0.471**		
				(0.216)		
sigma	Constant	0.00779***	0.00501***	0.00357***	0.00724***	0.00591***
	(0.00164)	(0.000846)		(0.000801)	(0.00154)	(0.00178)
	Observations	16	16	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 7: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	CO	CT	DC	DE	FL
ARMA	L.ar	0.665***	0.334	-0.707**		0.960***
		(0.253)	(0.295)	(0.332)		(0.122)
	L2.ar	-0.614***	-0.492**	-0.166		-0.732***
		(0.217)	(0.237)	(0.347)		(0.122)
sigma	Constant	0.00418***	0.00605***	0.00283***	0.00666***	0.00467***
		(0.00106)	(0.00105)	(0.000832)	(0.00115)	(0.00110)
	Observations	16	16	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 8: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	GA	HI	IA	ID	IL
ARMA	L.ar	0.566**			0.374	0.488
		(0.231)			(0.321)	(0.302)
	L2.ar	-0.623***			-0.489**	-0.605***
		(0.211)			(0.214)	(0.215)
sigma	Constant	0.00486***	0.00644***	0.00687***	0.00793***	0.00363***
		(0.000892)	(0.00149)	(0.00161)	(0.00181)	(0.000832)
	Observations	16	16	16	16	16

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 9: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	IN	KS	KY	LA	MA
ARMA	L.ar		0.277		0.469**	
			(0.211)		(0.216)	
	L2.ar	-0.385*	-0.564**		-0.524**	
		(0.221)	(0.239)		(0.222)	
sigma	Constant	0.00750***	0.00417***	0.00611***	0.00755***	0.00511***
		(0.00139)	(0.00109)	(0.000774)	(0.00196)	(0.000794)
	Observations	16	16	16	16	16

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 10: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	MD	ME	MI	MN	MO
ARMA	L2.ar			-0.428*		
				(0.227)		
sigma	Constant	0.00274***	0.00309***	0.00925***	0.00522***	0.00397***
		(0.000299)	(0.000439)	(0.00155)	(0.000903)	(0.000550)
	Observations	16	16	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 11: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	MS	MT	NC	ND	NE
ARMA	L.ar			0.697***		-0.482**
				(0.232)		(0.193)
	L2.ar			-0.619**		-0.705***
				(0.247)		(0.159)
sigma	Constant	0.00500***	0.00468***	0.00443***	0.00656***	0.00236***
		(0.00102)	(0.000673)	(0.00108)	(0.00139)	(0.000809)
	Observations	16	16	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 12: **Estimates of $L_i(\epsilon_{it})$ For Various States**

		NH	NJ	NM	NV	NY
EQUATION	VARIABLES					
ARMA	L.ar				0.876***	0.587***
					(0.186)	(0.157)
	L2.ar				-0.675***	-0.682***
	L3.ar	-0.465**				
		(0.232)				
					(0.152)	(0.165)
sigma	Constant	0.00457***	0.00431***	0.00450***	0.00727***	0.00409***
		(0.00100)	(0.000708)	(0.000760)	(0.00110)	(0.00108)
	Observations	16	16	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 13: **Estimates of $L_i(\epsilon_{it})$ For Various States**

		OH	OK	OR	PA	RI
EQUATION	VARIABLES					
ARMA	L.ar		0.0251	-0.0783	0.132	
			(0.714)	(0.226)	(0.344)	
	L2.ar		-0.129		-0.513*	
			(0.350)		(0.263)	
	L3.ar		-0.560*	-0.526**		
			(0.310)	(0.243)		
sigma	Constant	0.00670***	0.00343***	0.00643***	0.00325***	0.00422***
		(0.000952)	(0.000828)	(0.00145)	(0.000615)	(0.00101)
	Observations	16	16	16	16	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 14: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	SC	SD	TN	TX	UT
ARMA	L.ar	0.299		0.225		0.859***
		(0.483)		(0.233)		(0.206)
	L2.ar	-0.559**				-0.646***
		(0.248)				(0.197)
	L3.ar			-0.633**		
				(0.320)		
sigma	Constant	0.00443***	0.00487***	0.00398***	0.00564***	0.00424***
		(0.000721)	(0.00103)	(0.00145)	(0.00117)	(0.00102)
	Observations	16	16	16	16	16

Standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 15: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	VA	VT	WA	WI	WV
ARMA	L.ar	0.391*		0.646*	0.527***	
		(0.210)		(0.385)	(0.195)	
	L2.ar	-0.461**		-0.621**	-0.544***	
		(0.207)		(0.282)	(0.144)	
sigma	Constant	0.00352***	0.00507***	0.00600***	0.00371***	0.00409***
		(0.000824)	(0.000512)	(0.00172)	(0.000742)	(0.000757)
	Observations	16	16	16	16	16

Standard errors in parentheses

Table 16: **Estimates of $L_i(\epsilon_{it})$ For Various States**

EQUATION	VARIABLES	WY
c_var52	Constant	0.00785***
		(0.00117)
	Observations	16

Standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 17: **Estimate of $L_i(\epsilon_{it})$ For the State of Wyoming**

State	0	$-\sigma/2$	$-\sigma$	$-3/2\sigma$	-2σ
AL	0.5000	0.5074	0.5149	0.5223	0.5298
AK	0.5000	0.5050	0.5099	0.5149	0.5198
AZ	0.5000	0.5078	0.5156	0.5233	0.5311
AR	0.5000	0.5107	0.5214	0.5321	0.5428
CA	0.5000	0.5449	0.5891	0.6323	0.6739
CO	0.5000	0.5136	0.5272	0.5407	0.5542
CT	0.5000	0.5037	0.5073	0.5110	0.5147
DE	0.5000	0.5040	0.5079	0.5119	0.5158
DC	0.5000	0.5058	0.5115	0.5173	0.5230
FL	0.5000	0.5153	0.5306	0.5458	0.5609
GA	0.5000	0.5039	0.5077	0.5116	0.5155
HI	0.5000	0.5040	0.5080	0.5121	0.5161
ID	0.5000	0.5043	0.5086	0.5129	0.5172
IL	0.5000	0.5078	0.5155	0.5233	0.5311
IN	0.5000	0.5028	0.5057	0.5085	0.5113
IA	0.5000	0.5032	0.5064	0.5095	0.5127
KS	0.5000	0.5075	0.5151	0.5226	0.5301
KY	0.5000	0.5057	0.5113	0.5170	0.5227
LA	0.5000	0.5049	0.5099	0.5148	0.5197
ME	0.5000	0.5042	0.5084	0.5127	0.5169
MD	0.5000	0.5073	0.5146	0.5219	0.5292
MA	0.5000	0.5051	0.5103	0.5154	0.5206
MI	0.5000	0.5025	0.5049	0.5074	0.5099
			...		

Table 18: Probability That State i Dives For A Given Dive in Texas and California

State	0	$-\sigma/2$	$-\sigma$	$-3/2\sigma$	-2σ
			...		
MN	0.5000	0.5038	0.5075	0.5113	0.5151
MS	0.5000	0.5049	0.5098	0.5147	0.5196
MO	0.5000	0.5073	0.5146	0.5219	0.5292
MT	0.5000	0.5084	0.5169	0.5253	0.5337
NE	0.5000	0.5135	0.5269	0.5404	0.5537
NV	0.5000	0.5142	0.5283	0.5424	0.5565
NH	0.5000	0.5136	0.5271	0.5406	0.5541
NJ	0.5000	0.5086	0.5171	0.5257	0.5343
NM	0.5000	0.5105	0.5211	0.5316	0.5421
NY	0.5000	0.5075	0.5151	0.5226	0.5301
NC	0.5000	0.5097	0.5193	0.5289	0.5386
ND	0.5000	0.5050	0.5100	0.5150	0.5200
OH	0.5000	0.5033	0.5066	0.5098	0.5131
OK	0.5000	0.5130	0.5259	0.5388	0.5517
OR	0.5000	0.5070	0.5141	0.5211	0.5281
PA	0.5000	0.5090	0.5180	0.5270	0.5360
RI	0.5000	0.5052	0.5103	0.5155	0.5207
SC	0.5000	0.5065	0.5130	0.5195	0.5260
SD	0.5000	0.5030	0.5061	0.5091	0.5122
TN	0.5000	0.5098	0.5196	0.5294	0.5392
TX	0.5000	0.5647	0.6277	0.6875	0.7427
UT	0.5000	0.5087	0.5174	0.5261	0.5348
VT	0.5000	0.5041	0.5082	0.5122	0.5163
VA	0.5000	0.5083	0.5166	0.5249	0.5332
WA	0.5000	0.5140	0.5280	0.5419	0.5558
WV	0.5000	0.5151	0.5302	0.5453	0.5603
WI	0.5000	0.5222	0.5443	0.5662	0.5880
WY	0.5000	0.5063	0.5126	0.5190	0.5253

Table 18: Probability That State i Dives For A Given Dive in Texas and California

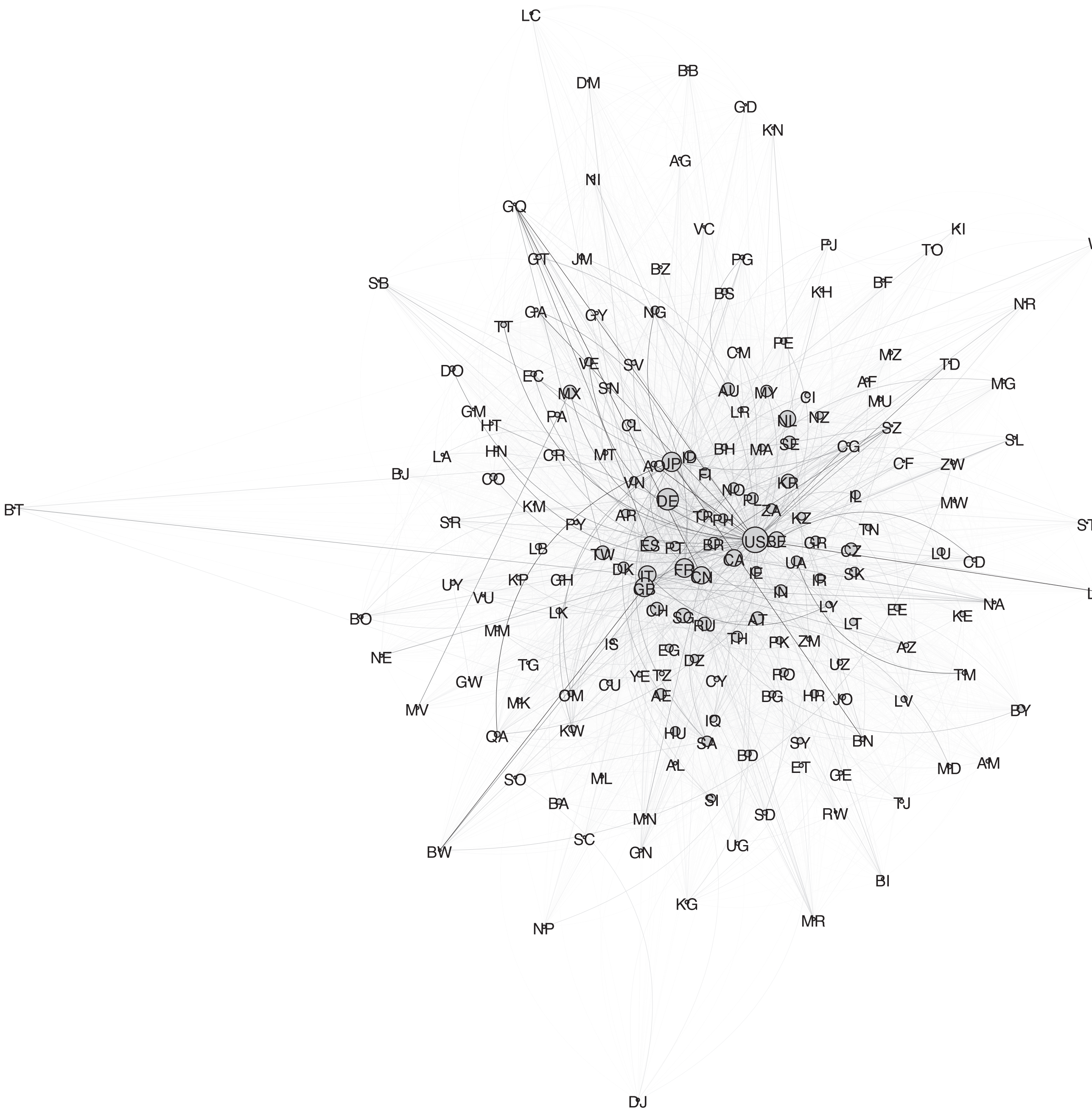
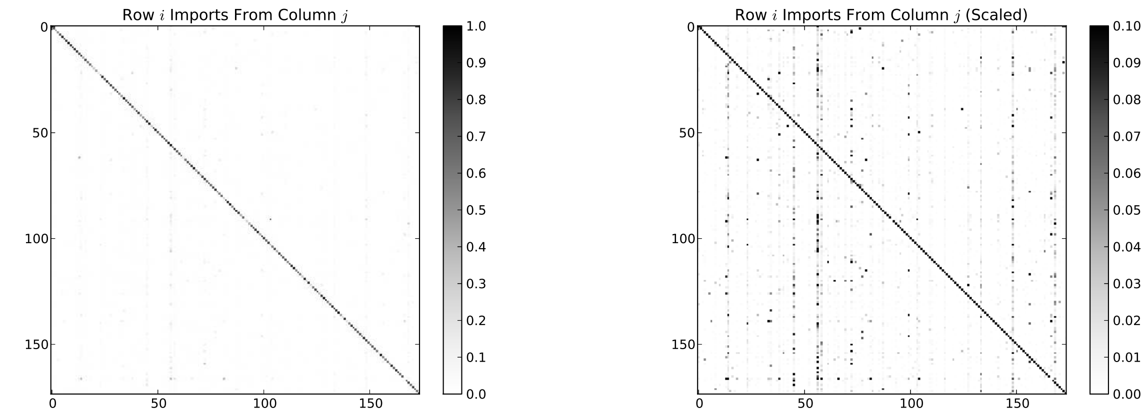


Figure 6: **Main Diagonal of** $(I-(1-\alpha)S)^{-1}(I-(1-\alpha)S)^{-1'}$ **(World Data)**

Source: Correlates of War Trade Data Set v.3.0. Network displayed according to the Fruchterman-Reingold layout. The repulsive force is proportional to the fourth root of production and a bounding box of 1000pts × 1000pts is used. Each node has a size proportional to the fourth root of production. The higher the share of trade, the darker are the edges. Representation produced with Python and iGraph.

Table 19: Parameter and Sources for the World Network

Symbol	Value	Calculation	Source
α	0.32	$\frac{wL}{Y}$	Ambler & Al. [2]
s_{ij}	N/A	$\frac{\text{average imports of pair } ij}{\sum_j \text{average imports of pair } ij}$	Correlates of War Dyadic Trade Dataset



(a) **Original Data** (b) **Colors Scaled**

Figure 7: **Heatmap of S' (World Dataset)**

Source: Correlates of War Trade Data Set v.3.0. Produced with Python, iGraph and PyPlot. In the second panel, the colorbar has been restricted between $[0, 0.1]$ to exhibit exporting countries.

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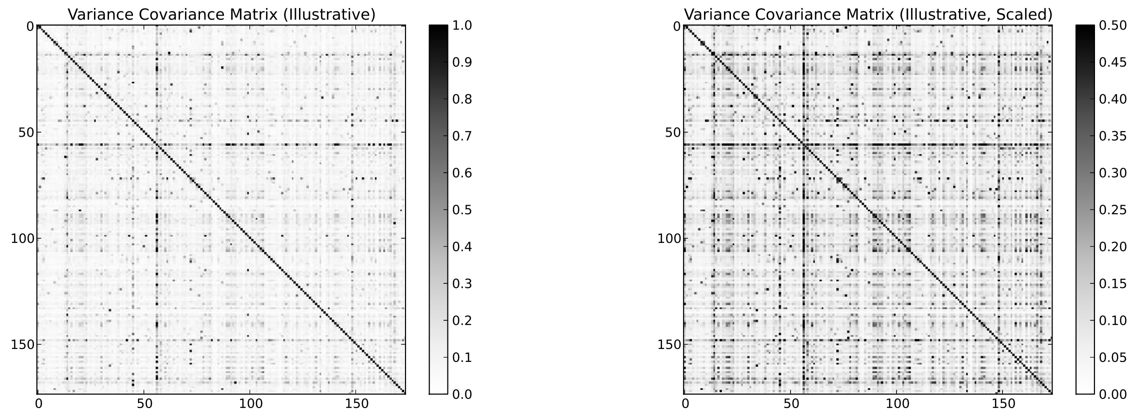

(a) **Original Data** (b) **Colors Scaled**

Figure 8: **Heatmap of** $(I-(1-\alpha)S)^{-1}(I-(1-\alpha)S)^{-1'}$ **(World Dataset)**

Source: Correlates of War Trade Data Set v.3.0. Produced with Python, iGraph and PyPlot. In the second panel, the colorbar has been restricted between $[0, 0.5]$ to exhibit covariance between countries.

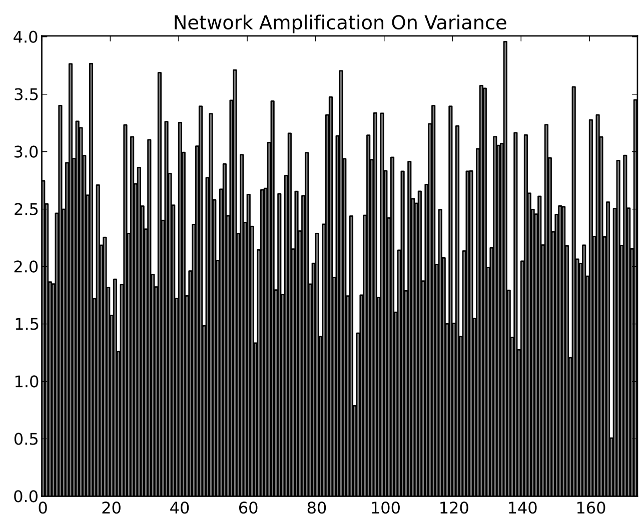

Figure 9: **Main Diagonal of** $(I-(1-\alpha)S)^{-1}(I-(1-\alpha)S)^{-1'}$ **(Variance)**

Source: Correlates of War Trade Data Set v.3.0. Produced with Python, iGraph and PyPlot.

Country i	N_{iGr}	Cumulative sum
Eurozone		
Greece	2.967	2.967
Cyprus	0.118	3.085
Germany	0.110	3.195
Italy	0.092	3.287
France	0.061	3.348
Bulgaria	0.050	3.398
United Kingdom	0.046	3.443
Netherlands	0.044	3.487
Malta	0.040	3.527
Belgium	0.037	3.564
Luxembourg	0.034	3.598
Spain	0.029	3.627
Slovenia	0.029	3.656
Austria	0.028	3.684
Czech Republic	0.027	3.711
Slovakia	0.025	3.735
Sweden	0.023	3.758
Ireland	0.022	3.781
Romania	0.022	3.802
Hungary	0.022	3.824
Croatia	0.021	3.845
Denmark	0.020	3.866
Finland	0.018	3.883
Lithuania	0.018	3.901
Poland	0.017	3.918
Estonia	0.017	3.934
Portugal	0.014	3.949
Latvia	0.014	3.963
ROW	1.857	5.820

Table 20: Cumulative Signifcance of Imports in the Eurozone

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